## TOPAS Version 5 - What's New

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## 2 New kernel functionality

### 2.1 Convolution

### 2.1.1.....ft_conv

The keyword ft_conv describes a Fourier Transform (FT) that is convoluted into phase peaks using a Fast Fourier Transform (FFT); for example, to convolute a Voigt into a phase the following can be used:

```
ft_conv = Exp(-(Pi FT_K gfwhm)^2 / (4 Ln(2)) - Pi FT_K lfwhm);
    ft_min = le-8; ' this is the default and is optional
    ft_x_axis_range = 40 lfwhm;
    , \overline{Ge\overline{t}}(ft\overline{0})
    , FT_Bre\overline{a}k
```

The convolution theorem is used here multiplying the FT of a Gaussian by the FT of a Lorentzian. Were the Fourier transforms separately defined then the program will internally use the convolution theorem.
FT_K is a reserved parameter name and it returns the transform $k$ divided by the $x$-axis range of the peak; this range includes ft_x_axis_range.
ft_x_axis_range can be an equation that needs to be set such that the transform decays to near zero; peak tails will otherwise be incorrect. A Lorentzian for example needs a large ft_x_axis_range for accurate x -axis tails.
ft_min defines the smallest value to which the transform is calculated to. For example, an already broadened peak in $x$-axis space will have a relatively narrow transform; the calculation of the transform is therefore terminated when $\mathrm{FT}(\mathrm{k}) / \mathrm{FT}(\mathrm{k}=0)$ <ft_min. Transform values for larger k are then set to zero. If $($, ,) constructs can instead be used within the transform equation for further control; for example:

```
ft_conv = If (FT_K > D, FT_Break, Sphere(FT_K, D));
```

Here the calculation of the FT is terminated when $\mathrm{FT} \_\mathrm{K}>\mathrm{D}$ using $\mathrm{FT} \_$Break.
Get(ft_0) returns $\mathrm{FT}(\mathrm{k}=0)$ and can be used within the ft_conv equation, for example,

```
ft_conv = {
    def a = Exp(-Pi FT_K lf);
    return If(a < le-6-
    }
```

$f t$ conv integrates with convolutions that are performed in direct space. It can be used within peak stack operations and it can be a function of the reserved parameter names:

H, K, L, M, Th, Xo, D_spacing, FT_K and spherical_harmonics_hkl

Multiple ft_conv (s) can be defined at either the $x d d$ or phase level. When defined at the $x d d$ level the convolution is applied to all phases of that $x d d$.

The test_examples|ft directory comprises examples that use ft_conv. For a typical Rietveld refinement, an ftconv used to describe a Voigt is almost as fast as the analytical equivalent as seen in example ftlalvo4a.inp. For high accuracy the range of the peak, as defined with ft_x_axis_range, needs to be large, up to 400 FWHM for a Lorentzian; in these cases the ft_conv is considerably slower as seen in ftlvoigt.inp.
ftlalvo4a.inp compares spherical_harmonics_hkl used with and without ft_conv.

```
prm csl 50 min 3 max = Min(Val 2 + .1, 10000);
prm csg 50 min 3 max = Min(Val 2 + .1, 10000);
prm csl_fwhm = 0.1 Rad Lam / (csl Cos(Th));
```

```
prm csg_fwhm = 0.1 Rad Lam / (csg Cos(Th));
if 1 {
    ' Spherical Harmonics
    spherical_harmonics_hkl sh
                sh_order 2
                loād sh_Cij_prm
                {
                    y00 !sh c00 1
                    y20 sh_\overline{c}20 0
                    y21p sh_c21p 0
                    y21m sh_c21m 0
                    y22p sh_c22p 0
                    y22m sh_c22m 0
        }
    existing_prm csl_fwhm *= sh;
    existing_prm csg_fwhm *= sh;
}
if 0 {
    ' use analytical Lorentzian and Gaussian convolution
    lor_fwhm = csl_fwhm;
    gauss_fwhm = c\overline{Sg_fwhm;}
} else {
    ' use Fourier Transform convolution
    ft_conv = Exp(-(Pi FT_K csg_fwhm)^2 / (4 Ln(2)) - Pi FT_K csl_fwhm);
        ft_x_axis_range = 4 5 csl_fwhm + 4 csg_fwhm;
}
```

The speed of the analytical convolution is greater not simply because describing the peak analytically is faster but because derivatives of multiple parameters for lor_fwhm (or gauss_fwhm) requires only one peak calculation; whereas for ft_conv the peak is recalculated for each independent parameter that it is a function of.

### 2.1.2.....ft_conv compared to user_defined_convolution

If a response function is known in x -axis space then it is often best to perform the convolution in x -axis space rather that describing the FT using ft_conv. The keyword user_defined_convolution can be used to perform convolution in $x$-axis space and the speed at which it operates is as fast or faster than ft_conv depending on the x -axis range of the response function; this is demonstrated in ftlorentzian.inp. For each peak user_defined_convolution estimates the computational effort required to perform the convolution directly and with a FFT and chooses the one with the least computational effort. Examples that use user_defined_convolution are as follows:

```
ft\lorentzian.inp
tof\tof_bank2_2.inp
wppm\gamma.inp
udefa.inp
```

udefa.inp in particular shows how to convolute a function with discontinuities in x-axis space; ie.

```
user_defined_convolution = Exp(-20 X^2); min = -. 2; max = . 5;
```

The FT for functions with such discontinuities often cannot be described analytically and hence the usefulness of user_defined_convolution.

### 2.1.3.....FFT versus direct convolution

Typically a FFT convolution is quoted as comprising $\mathrm{O}\left(\mathrm{N} \log _{2} \mathrm{~N}\right.$ ) operations (Cooley-Tukey algorithm for example) versus a direct convolution that comprises $\mathrm{O}\left(\mathrm{N}^{2}\right)$ operations, see https://ccrma.stanford.edu/~jos/ReviewFourier/FFT_Convolution_vs_Direct.html for example, and that direct convolution is only faster for $\mathrm{N}<128$. However, in XRD work a direct convolution rather than an FFT working on real numbers is often faster for $N \sim<256$ to 512 as the comparison of the $\mathrm{O}\left(\mathrm{N} \log _{2} N\right)$ versus $\mathrm{O}\left(\mathrm{N}^{2}\right)$ is invalid. To see why consider a response function comprising 3 points and a peak comprising 5 points. A convolution can be pictured as the response function $R$ moving along the peak $P$ as follows:

```
0
    - - x
        - x x
        x X X
            x X X
                x x x
                x X -
                    x - -
```

In this representation each ' $x$ ' can be considered a multiply and in direct convolution this makes a total of 15 multiplies ( $\mathrm{Nr}{ }^{*} \mathrm{~Np}$ ) and not $\mathrm{N}^{2}$ where $\mathrm{N} / 2 \leq(\mathrm{Nr}+\mathrm{Np}) \leq \mathrm{N}$. To perform such a convolution with an FFT the number of operations is approximately $4^{*} 16^{*} \log _{2} 16=256$ multiplies where 16 is the closest power of 2 to $\mathrm{Nr}+\mathrm{Np}$. Of course FFT routines typically also have special cases for small $N$; nonetheless $N=256$ to 512 is not small and many peaks in XRD work typically comprise less points and in particular many of the response functions have a small Nr ; these include axial divergence, equatorial divergence, receiving slit width, capillary convolution, LPSD convolution and often sample penetration.

### 2.1.4..... Convolutions in general

TOPAS approximates the number of operations required for direct and FFT convolution and chooses the one with the smaller number of operations. In addition all direct convolutions are performed with peaks treated as straight line segments. Response functions are either straight line segments, analytical or both. The extra cost of the piece wise integration is small, approximately $3(\mathrm{Nr}+\mathrm{Np})$ operations, and the benefit is a high degree of accuracy.

Apart from lor_fwhm and gauss_fwhm, all of the convolutions described below have discontinuities in 2Th space; their associated Fourier transform therefore is difficult to describe. In cases where a FT is less demanding then a FFT is used after first calculating the aberration in 2Th space.

Response functions that are treated as straight line segments are:

```
user_defined_convolution
capil}lary_diāmeter_mm
lpsd_th2_āngular_rānge_degrees
```

Response functions that are analytically convoluted with the straight line segments of the peak are:

```
exp_conv_const
hat
stacked_hats_conv
```

Response functions that comprise a mixture of analytical and straight line segments are:

```
axial_conv
one_on_x_conv
circles_conv
```

lor_fwhm and gauss_fwhm convolutions are treated analytically with the emission profile to form the base profile. Convolutions are calculated with a step size given by:

```
Peak_Calculation_Step = x_calculation_step / convolution_step
```

For efficiency $x$ _calculation_step should not be defined for data with equal x-axis steps; instead rebin_with_dx_of should be used. The following response functions are calculated at smaller step sizes without changing Peak_Calculation_Step or Nr:

```
axial_conv, Step = Peak_Calculation_Step / 2
lpsd_th2_angular_range_degrees, Step = Peak_Calculation_Step / 3
capi\overline{l}lary_diameter_mm, Step = Peak_Calculation_Step / 1
```

In this manner a high degree of accuracy is maintained for the little extra cost in calculating the extra response function points and with the benefit of not increasing Np*Nr. Typically a laboratory diffraction pattern can be accurately synthesized with a Peak_Calculation_Step of 0.02 degrees 2 Th . The next step to increasing accuracy would be to increase convolution_step to 2 and so on.

When direct convolution is used then most convolutions scale by ( Nr * Np ) except for convolutions that scale by N ; these are always performed directly and they are:

```
exp_conv_const
hat
stacked_hats_conv
```

Calculating derivatives of parameters that are a function of a convolution can be demanding. Most convolutions however that have multiple dependent parameters require only one recalculation of the convolution; exceptions are ft_conv, WPPM_conv and user_defined_convolution. In the case of convolutions that comprise multiple convolution parameters, for example, axial_conv with its convolution parameters of primary_soller_angle etc..., then a recalculation for each of the convolution parameters is required.

The following is an overview of the convolution and the aberration that uses it:

| axial_conv | Full Axial divergence model |
| :--- | :--- |
| one_on_x_conv | Equatorial Divergence |
| circles_conv | Simple axial model |
| capillary_diameter_mm | Capillary sample |
| lpsd_th2_angular_range_degrees | LPSD detector |
| exp_conv_const | Sample penetration with or without a finite thickness |
| hat | Receiving slit width, sample tilt |
| stacked_hats_conv | Tube tails |

### 2.2 WPPM

Examples referred to in this section reside in the test_examples\wppm directory.

### 2.2.1 ....... WPPM in 2Th space

The WPPM microstructure analysis (Scardi \& Leoni, 2001; Leoni et al. 2004; David et al. 2010) for domains comprising spheres and a gamma distribution can be implemented using user_defined_convolution operating in 2Th space as shown in gamma.inp.

### 2.2.2 ....... WPPM using fit_obj(s)

For cases where microstructure broadening is far greater than instrument/emission profile broadening then fit_obj's can be used to describe the peak shape (see gamma-fit-obj.inp and sphere-fit-obj.inp), for example:

```
fn gamma_mu_variance(mu, v, xo)
    {
        def s = 2 ( Sin( X Pi/360) - Sin(xo Pi/360) ) / lam;
        def p0 = Pi s mu;
        def p = If(Abs(p0) < 1e-10, 1, p0);
        def q = 2 p / v;
        return
            mu v / p^4
            (
                2 p^2 / (2 + v)
            +(v / (2 + 3 v + v^2)) (1 - (1 + q^2)^(-.5 v) Cos(v ArcTan(q))
            -2 p (1 + q^2)^(-.5 (v+1)) Sin( (1 + v) ArcTan(q)))
        );
    }
```

Example super-lorentzian.inp is useful for asking the question; can spheres with a gamma distribution describe a $1 /\left(1+x^{\wedge} 2\right)^{\wedge} m$ type function?

Example compare-1.inp is useful for asking the question; can a Voigt fit to a particular case of spheres with a gamma distribution?

### 2.2.3 ....... WPPM using WPPM_ft_conv

WPPM_ft_conv describes a FT in s space and performs a convolution on phase peaks that have been interpolated to $s$ space, for example:

```
WPPM_ft_conv = 1 - 1.5 WPPM_L / D + 0.5 (WPPM_L / D)^3;
    WPPM L max = D;
    WPPM_t\overline{h}2_range = 25 . 1 Rad Lam / (D Cos(Th));
    WPPM_correct_Is
```

The result is then interpolated back to 2Th space. Interpolations are scaled such that $\mathrm{I}(s) d s=\mathrm{I}(\theta) d \theta$ when WPPM_correct_Is is defined; the affects of this scaling is typically small at low angles and becomes noticeable at very high angles reaching a maximum at 180 degrees 2 Th where the derivative of $\mathrm{Cos}(\mathrm{Th})$ is at a maximum.

When multiple WPPM_ft_conv(s) are defined then the program will internally use the convolution theorem.
WPPL_L is a reserved parameter name that returns the transform parameter.
WPPM_L_max defines the maximum WPPL_L.
Get(ft_0) and FT_Break can both be used in WPPM_ft_conv in a manner similar to ft_conv.
The tails of WPPM peaks extend for almost the whole diffraction pattern; they can be shortened using WPPM_th2_range; in the above example this range has been written in terms of the fwhm as defined in the Scherrer equation.

WPPM_ft_conv can be a function of the following reserved parameter names:
H, K, L, M, Th, Xo, D_spacing, WPPM_L and spherical_harmonics_hkl

Example s-sphere-1.inp uses WPPM_ft_conv to fit to a synthesized WPPM generated peak with identical results.

The following macros (written by Matteo Lenoi), as defined in TOPAS.INC, describes a log normal distribution:

```
WPPM Cube Ln Normal
WPPM-Spherre \overline{Ln_Normal}
WPPM_Octahedron_Ln_Normal
```

Where for example WPPM_Octahedron_Ln_Normal is as follows:

```
macro WPPM_Octahedron_Ln_Normal(muc, muv, sigc, sigv)
```

```
{
    #m_argu muc
    #m_argu sigc
    If_Prm_Eqn_Rpt(muc, muv, min . 1 max = Min(2 Val + . 3, 100);)
    If_Prm_Eqn_Rpt(sigc, sigv, min . O1 max = Min(2 Val + . 01, 3);)
```



```
        def cga = Constant(Cos(Get(ga) Pi/180));
        def sga = Constant(Sin(Get(ga) Pi/180));
        def cal = Constant(Cos(Get(al) Pi/180));
        def cbe = Constant(Cos(Get(be) Pi/180));
        def aa = Constant(Get(a));
        def b.b = Constant(Get(b));
        def cc = Constant(Get(c));
        def cv = Constant(Get(cell_volume));
        def wA = H / aa;
        def wB = (-H cga / aa + K / bb) / sga;
        def wC = ( (H bb cc (cal cga - cbe) + K aa cc (cbe cga - cal)) /
            sga + L aa bb sga) / cv;
        def A = D_spacing Max(wA, wB, wC);
        def B = D_spacing Max(Min(wA, wB), Min(wA, wC), Min(wB, wC));
        def C = D_-spacing Min(wA, wB, wC);
        def HO = 1;
        def H1 = If (A>=B+C,-3 A/Sqrt(2),- 3 (A + B + C)/Sqrt(8));
        def H2 = If (A>=B+C, 3 (A A - B B - C C)/2,-3(A A+(B-C)^2 - 2 A
            (B+C)) / 4);
        def H3 = If (A>=B+C, (-A^3 + 3 A (B B+C C) + 2(B^3 + C^3))/(2
                Sqre(2)),(A^3 + B^3 + C^^3-3 A B C)/Sqrt (2));
        def Kc = (A + B + C)/Sqrt(2);
        def u = CeV(muc, muv);
        def sig = CeV(sigc, sigv);
        fn M(n) = Exp(Min(n u + 0.5 n^2 sig, 600));
        fn wppm_Ln(kc) = Get(WPPM_Ln_k) + Ln(kc Get(WPPM_dL));
        fn q(Hn, n) {
        return
            Hn
                Erfc_Approx( ( wppm_Ln(Kc) - u - (3-n) sig^2) / (sig
                    Sqrit(2)))
                WPPM_L^n
                M (3-\overline{n}) ;
        }
    return q(H0, 0) + q(H1, 1) + q(H2, 2) + q(H3, 3);
    }
    WPPM_break_on_small = 1e-7;
    WPPM-L_max }\mp@subsup{}{}{-}10\overline{0}
    WPPM_th2_range = 30;
}
```

Example cube-In-normal-1.inp can be used to test these macros. Lattice parameters appearing within the macros are made constant using Constant; thus these convolutions are made independent of lattice parameter changes and hence a separate convolution is not initiated whilst calculating lattice parameter derivatives.

WPPM_Ln_k is a reserved parameter name that returns Ln of an integer and is used to calculate Ln(Kc WPPM_L) in a fast manner.

The example In-normal-1.inp can be used for visualizing a Ln normal distribution. It uses the Ln_Normal_x_at_CD function to determine the limit of the distribution.

### 2.3 Loading of INP files

### 2.3.1..... if $\}$ else if $\}$ else $\}$

An 'if' construct operational on the loading of INP files, see test_examples\zro2.inp. Loading operates on the pre-processed INP file; syntax is as follows:

```
if expression {
} else if expression {
} else expression {
}
```

expression can be any valid TOPAS equation without the semicolon; in addition expression can contain the functions Prm_There(prm_name) and Obj_There(obj_name). The following is equivalent to a/* */ block comment:

```
if 0
    ...
}
```

A more complex construct could look something like the following:

```
xdd
    local aaa 1
    str...
        local aaa 2
        str...
        local aaa 3
    hkl Is
        if Prm There(aaa) {
            Out(aaa, "\nThis is the aaa at the xdd level %-1.6f")
            if aaa == 2 {
                Out_String("\nThis wont be written to file as aaa at the xdd
                                level is 1")
            }
        } else if Obj_There(hkl_Is) {
                Out_String("\nYes this is a hkl_Is phase")
        } else {
                        Out_String("\naaa is not there and this is not a hkl_Is phase")
        }
for xdds {
    if And(Obj_There(neutron), Obj_There(pk_xo)) {
        ' Neutron TOF
    }
}
```


### 2.3.2.....existing_prm

existing_prm allows for the modification of an existing prm/local parameter, see for example the macro K_Factor_WP in TOPAS.INC. The following:

```
local a 1
existing_prm a += 1;
existing_prm a /= 2;
existing_prm a = 3 (a + 1);
prm = a; : 0
```

will give the result:

```
prm = a; : 6.00000
```

The operators of $+=,-=,^{*}-, /=$ and ${ }^{\wedge}=$ are allowed.

### 2.4 CIF

The following macros and Get's can be used to output data in CIF format; Red corresponds to new macros:

```
Out_CIF_STR(file)
Out_CIF_ADPs(file)
Out+}\mp@subsup{}{}{-}\mp@subsup{ClF}{}{-}\mathrm{ STR(file, with id)
Out_CIF_Bonds_Angles(fíle)
Get\overline{(number of_parameters)}
Get(refine_ls_shift_on_su_max)
Get(weighting)
Xi = a reserved parameter name
```

_refine_ls_shift/su_max can be accessed using Get(refine_ls_shift_on_su_max) when do_errors is defined and when continue_after_convergence is NOT defined. A message similar to the following is displayed on calculation:

```
refine_ls_shift_on_su_max 0.409610469 corresponds to parameter m501b939c_3 of
object-
```

Get(weighting) and Xi can be used as follows:

```
xdd_out file append load out_record out_fmt out_eqn
    " %9.0f" = Xi;
    " %11.5f" = X;
    " %11.5f" = Ycalc;
    " %11.5f" = Yobs;
    " %11.5f\n" = Get(weighting);
    }
```

Get(weighting) returns the following masked with excluded regions:

```
1 / Max(1, Yobs), if SigmaYobs does not exist
1 / SigmaYobs^2, if SigmaYobs does exist
```

If weighting is a function of YCalc etc... then it returns the last weighting calculated depending on recal_weighting_on_iter.

### 2.5 Rigid bodies

### 2.5.1..... Rigid body parameter errors propagated to fractional coordinates

Errors for fractional coordinates for sites defined as part of a rigid body are now propagated to the site fractional coordinates. The example rigid-errors\Aniline_I_100K_x.inp (by Simon Parsons) demonstrates equivalent refinements for the case of 1) using a rigid body and for the case 2 ) hand coding the fractional coordinates in terms of rigid body parameters but not in fact using a rigid body. Errors and convergence behaviour in both cases are identical. In particular case (2), which has many computer algebra equations, takes approximately the same time per iteration as case (1); this demonstrates that the computer algebra does not noticeably affect computational speed even in cases where its use is plentiful.

### 2.5.2..... Z-matrix collinear error information

The Z-matrix collinear points exception can be deciphered using information displayed on detection of the error. The collinear error is due to three atoms on a z-matrix line which are collinear. The information displayed includes a snap shot of the rigid body operations pertaining to the error. The following is an example of the information displayed:

```
DB_x_CB Zero dot product - Z-matrix possible collinear points at atoms
```

```
            O10
            C16 8.91631604e-016 1.0912987e-014 5.2
            C15 3.72315026e-016 1.0912987e-014 3.9
            C11 0 0 0
Partial z-matrix in error:
    rigid
        z_matrix C11
        z_matrix C12 C11 1.3
        z_matrix C13 C12 1.3 C11 120
        z_matrix C14 C13 1.3 C12 120 C11 180
        z-matrix C15 C14 1.3 C13 120 C11 0
        z_matrix C16 C15 1.3 C14 120 C11 180
        z_matrix O10 C16 1 C15 108 C11 120
```

To investigate why the error is occurring the rigid body fragment can be copied to a Rigid-body editor window; ie.


The O10 line is commented out as it's the line in error. Looking at the O10 line (using the OpenGL window) it can be seen that atoms C16, C15, C11 lie on a straight line; this is invalid as it becomes impossible to form the dihedral angle in a non-degenerate manner. The best way to think about a z-matrix line with 4 atoms A, $B, C, D$, ie.

```
z_matrix A B # C # D #
```

is to think of two triangles $A B C$ and $D B C$ hinged along the line $B C$. The angle between the triangles is the dihedral angle. If $B, C, D$ are collinear then there's no triangle DBC and hence the dihedral angle cannot be formed. Thus for z-matrices both A,B,C and B,C,D must not be collinear. Dummy atoms can solve this problem. The program tests for a zero dot product numerically with a tolerance of $1.0 \mathrm{e}-15$.

### 2.6 Functions - fn, def, return, noinline

Functions can be defined using the 'fn' keyword; here's an example of a recursive function:

```
fn factorial(x) { return If(x == 1, 1, x factorial(x-1)); }
prm = factorial(5); : 120
```

There's also the simple form where the 'return' statement is implied:

```
fn factorial(x) = If(x == 1, 1, x factorial(x-1));
```

The equation part of 'prm' objects can have a function body (see the macro Robust_Refinement in TOPAS.INC), for example

```
prm = { def a = 2; return a; }
```

Most importantly functions can reference parameters defined with the 'prm' keyword; this simplifies the writing of 'prm' equations and additionally memory usage can be greatly reduced when the 'noinline' keyword is used. Equations called 'def' objects can be used and defined within non-simple functions. Here's an example:

```
fn gauss(a, x, f, g)
    {
        def al = 2 Sqrt(Ln(2) / Pi) / f;
        def a2 = 4 Ln(2);
        def a3 = (x / f);
        return a1 Exp(-a2 a3^2);
    }
```

A 'def' object must be defined prior to its use. They can be assigned to other 'def' objects but not to objects of 'prm' type. In other words 'prm' objects are write-protected within functions. The arguments to functions can be 'def' or 'prm' objects. c-style braces can be used to scope variables; the following will throw an exception due to the attempted use of an uninitialized 'def' object:

```
fn foo(x) { def a; { def a = x; } return a; }
prm = foo(3); : 0 ' Exception thrown
```

The following will not throw an exception as the simplification routines will recognize the ' 0 ':

```
fn a(x) = x undefined_name 0; prm = a(3); : 0
```

Functions can be nested; for example:

```
fn foo() {
    def a, b;
    a = 3; b = 2;
    fn nested(x, y) { return Sqrt(x^2 + y^2); }
    return nested(a, b);
}
prm = foo(); : 6
```

'def' and 'prm' objects have scope and their scope determine the actual object used.
Here def ' $a$ ' is returned:

```
fn a(a) { def a = 2; return a; } prm=a(1) : 2
```

Here prm ' $a$ ' is returned:

```
prm a = 2; fn a() = a; prm=a(); : 2
```

Here the argument ' $a$ ' is returned:

```
prm a = 2; fn a(a) = a; prm=a(3); : 3
```

Function specifics

- fn's are a kernel operation and not a pre-processor operation.
- fn's must be defined prior to their use.
- fn arguments are optional but parentheses must be used in both the function definition and its use.
- a fn cannot be defined with a name of a previously defined fn name.
- fn's are inlined by default.
- Non-nested fn's can be prevented from being inlined with the 'noinline' prefix.
- nested functions cannot be prefixed with 'noinline'

Use of noinline can often be slower than not using noinline as a stack mechanism is used for the fn arguments as well as the global simplification routines cannot simplify what's inside a noinlined function. Functions are therefore 'inlined' (the word 'expand' is sometimes used) by default. A macro can be considered an inlined function and there's no difference in the how the following is finally processed:

```
fn my_max(a, b, c) = Max(a, b, c);
macro & my_max(& a, & b, & c) { Max(a, b, c) }
```

The macro by definition is inlined in the pre-processed INP file. In the case of 'fn' the program will inline 'my_max'. Prefixing fn with 'noinline' prevents inlining, for example:

```
noinline fn gauss(x, f) = (2 Sqrt(Ln(2)/Pi)/f) Exp(-4 Ln(2)((X - x)/f)^2);
```

Its best to inline small functions as it gives the simplification routines a chance to simplify what's inside the function in regards to its surroundings. Consider the following:

```
noinline fn a(b, c) = b^2 + c^^2;
prm pl 1
prm !p2 1
prm p3 1
prm !p4 1
prm p5 =a(p1, p2) +a(p3, p4); : 0
```

Without inlining the simplification routines won't see that p2 and p4 are constants inside the 'a' function and hence no simplification is performed; the ' $a$ ' function will be called twice and the stack used twice. Note, stack here refers to the computer algebra stack. With inlining p5 after simplification reduces to:

```
prm p5 = p1^2 + p3^2 + 2;:0
```

In the case of large functions then not inlining may increase performance as the signalling of equation nodes for recalculation will be reduced. Inlined functions have scope allowing the use of the Get() function, for example:

```
fn lat(h, k, l) = h Get(a) + k Get(b) + l Get(c);
str..
    lor_fwhm = lat(H, K, L) - lat(-H, -K, -L);
```


### 2.6.1.....Subject independent single crystal refinement

The \functions\alvo4-fn.inp example performs a single crystal refinement using the computer algebra aspects of the program. No x-ray or subject dependent keywords have been used; instead only six keywords are utilized:
fn, noinline, def, return, prm, restraint
The speed of alvo4-fn.inp is 7.4 times slower than the comparable subject dependent keyword equivalent of alvo4-normal.inp. Much of the difference in speed is in the calculation of the Cosines necessary for the structure factors. Importantly convergence and the behaviour of the parameters are similar. The placement of noinline is important. Also used is the out_refinement_stats keyword which outputs the following:

```
First pass equation statistics excluding attribute equations
    Number of equations : 534
    Number of nodes : 99751
    Number of nodes if expanded : 12070283
```

```
Number of penalties/restraints: 532
Number of independent penalty/restraints parameters: 58
Number of penalties/restraints: 532
Number of independent penalty/restraints parameters: 58
Time 0.13
Second pass equation statistics excluding attribute equations
    Before/After equation simplification
        Number of equations : 549 553
        Number of nodes : 99766 8354
        Number of nodes if expanded : 12070298 228183
Number of objects taking part in refinement: 73
Number of dependent parameters with derivatives wrt to Ycalc: 15
```

The alvo4-fn.inp demonstrates the ease at which an entire single crystal refinement can be performed; it should allow for user defined temperature factors etc...

### 2.6.2..... Computer algebra and out_refinement_stats

The computer algebra system CAS in version 5 (Coelho et al., 2011) is around 2 to 4 times faster than version 4; compare with running ROSENBROCK-10.INP or PVS.INP. The CAS has been reworked and it now operates on a global level where equations are simplified across all objects. The out_refinement_stats keywords, for SERINE_I_EVANS_N_TA_BANG_ROT.INP for example, outputs the following equation statistics:

```
Second pass equation statistics excluding attribute equations
    Before/After equation simplification
        Number of equations : 2707 3085
        Number of nodes : 22941 16671
        Number of nodes if expanded : 1706390373 1070170132
Number of objects taking part in refinement: 2595
Number of dependent parameters with derivatives wrt to Ycalc: 2319
```


### 2.7The Minimization Routines

The Newton-Raphson non-linear least squares method is used by default with the Marquardt method (1963) included for stability. A Bound Constrained Conjugate Gradient (BCCG) method (Coelho, 2005) incorporating $\mathrm{min} / \mathrm{max}$ limits is used for solving the normal equations. The objective function $\chi^{2}$ is written as:

$$
\begin{gather*}
\chi^{2}=\chi_{0}^{2}+\chi_{\mathrm{P}}^{2}+\chi_{\mathrm{R}}^{2}  \tag{2-1}\\
\text { where } \chi_{0}^{2}=\mathrm{K} \sum_{\mathrm{m}=1}^{\mathrm{M}} \mathrm{w}_{\mathrm{m}}\left(\mathrm{Y}_{\mathrm{o}, \mathrm{~m}}-\mathrm{Y}_{\mathrm{c}, \mathrm{~m}}\right)^{2}  \tag{2-2}\\
\chi_{\mathrm{P}}^{2}=\mathrm{K} \mathrm{~K}_{1} \mathrm{~K}_{\mathrm{P}} \sum_{\mathrm{p}=1}^{\mathrm{N}_{\mathrm{p}}} \mathrm{P}_{\mathrm{p}} \\
\chi_{\mathrm{R}}^{2}=\mathrm{K} \mathrm{~K}_{1} \mathrm{~K}_{\mathrm{R}} \sum_{\mathrm{r}=1}^{\mathrm{N}_{\mathrm{R}}} \mathrm{R}_{\mathrm{r}}^{2} \\
\mathrm{~K}=1 / \sum_{\mathrm{m}=1}^{\mathrm{M}} \mathrm{w}_{\mathrm{m}} \mathrm{Y}_{\mathrm{o}, \mathrm{~m}}^{2} \tag{2-3}
\end{gather*}
$$

$Y_{o, m}$ and $Y_{c, m}$ are the observed and calculated data respectively at data point $m, M$ the number of data points, $\mathrm{w}_{\mathrm{m}}$ the weighting given to data point m which for counting statistics is given by $\mathrm{w}_{\mathrm{m}}=1 / \sigma\left(\mathrm{Y}_{0, \mathrm{~m}}\right)^{2}$ where $\sigma\left(\mathrm{Y}_{0, \mathrm{~m}}\right)$ is the error in $Y_{o, m}, P_{p}$ are penalty functions, defined using the keyword penalty, and $N_{p}$ the number of penalty
functions. $R_{r}$ are restraints, defined using the keyword restraint, and $N_{r}$ the number of restraints. $K_{P}$ and $K_{R}$ are weights applied to the penalty functions and restraints respectively. $\mathrm{K}_{1}$ corresponds to the user defined penalties_weighting_K1 (default value of 1), typical values range from 0.1 to 2 . Penalty functions and Restraints are minimized when there are no observed data $Y_{0}$; see example ONLYPENA.INP.

The normal equations are generated by the usual expansion of $Y_{c, m}$ to a first order Taylor series around the parameter vector $\mathbf{p}$. The size of $\mathbf{p}$ corresponds to the number of independent parameters N . The penalty functions are expanded to a second order Taylor series around the parameter vector $\mathbf{p}$. The restraints are expanded to a first order Taylor series around the parameter vector $\mathbf{p}$. The resulting normal equations are:

$$
\begin{equation*}
\mathbf{A} \Delta \mathbf{p}=\mathbf{Y} \tag{2-4}
\end{equation*}
$$

$$
\begin{gather*}
\text { where } \mathbf{A}=\mathbf{A}_{0}+\mathbf{A}_{\mathrm{P}}+\mathbf{A}_{\mathrm{R}} \\
\mathbf{Y}=\mathbf{Y}_{0}+\mathbf{Y}_{\mathrm{P}}+\mathbf{Y}_{\mathrm{R}} \\
\mathrm{~A}_{0, \mathrm{ij}}=\sum_{\mathrm{m}=1}^{\mathrm{M}} \mathrm{w}_{\mathrm{m}} \frac{\partial \mathrm{Y}_{\mathrm{c}, \mathrm{~m}}}{\partial p_{\mathrm{i}}} \frac{\partial \mathrm{Y}_{\mathrm{c}, \mathrm{~m}}}{\partial p_{\mathrm{j}}}  \tag{2-5}\\
\mathrm{~A}_{\mathrm{P}, \mathrm{ij}}=\mathrm{K}_{\mathrm{P}} \frac{1}{2} \sum_{\mathrm{p}=1}^{\mathrm{N}_{\mathrm{p}}} \frac{\partial^{2} \mathbf{P}_{\mathrm{P}}}{\partial p_{\mathrm{i}} \partial p_{\mathrm{j}}} \\
\mathrm{~A}_{\mathrm{R}, \mathrm{ij}}=\mathrm{K}_{\mathrm{R}} \sum_{\mathrm{r}=1}^{\mathrm{N}_{\mathrm{R}}} \frac{\partial \mathrm{R}_{\mathrm{r}, \mathrm{i}}}{\partial p_{\mathrm{i}}} \frac{\partial \mathrm{R}_{\mathrm{r}, \mathrm{j}}}{\partial p_{\mathrm{j}}} \\
\mathrm{Y}_{0, \mathrm{i}}=\sum_{\mathrm{m}=1}^{\mathrm{M}} \mathrm{~W}_{\mathrm{m}}\left(\mathrm{Y}_{\mathrm{o}, \mathrm{~m}}-\mathrm{Y}_{\mathrm{c}, \mathrm{~m}}\right) \frac{\partial \mathrm{Y}_{\mathrm{c}, \mathrm{~m}}}{\partial p_{\mathrm{i}}} \\
\mathrm{Y}_{\mathrm{P}, \mathrm{i}}=-\frac{\mathrm{K}_{\mathrm{P}}}{2} \sum_{\mathrm{p}=1}^{\mathrm{N}_{\mathrm{P}}} \frac{\partial \mathrm{P}_{\mathrm{P}}}{\partial p_{\mathrm{i}}} \\
\mathrm{Y}_{\mathrm{R}, \mathrm{i}}=-\mathrm{K}_{\mathrm{R}} \sum_{\mathrm{r}=1}^{\mathrm{N}_{\mathrm{R}}} \mathrm{R}_{\mathrm{r}} \frac{\partial \mathrm{R}_{\mathrm{r}}}{\partial p_{\mathrm{i}}}
\end{gather*}
$$

The Taylor coefficients $\Delta \mathbf{p}$ correspond to changes in the parameters p. Eq. (2-4) represents a linear set of equations in $\Delta \mathbf{p}$ that are solved for each iteration of refinement. Off diagonal terms in $\mathbf{A}_{P}$ are not calculated and are instead set to zero.
$\mathrm{K}_{\mathrm{R}}$ and $\mathrm{K}_{\mathrm{P}}$ are both set to 1 in the absence of $\chi_{0}^{2}$. When $\chi_{0}^{2}$ does exist then $\mathrm{K}_{\mathrm{P}}$ is used to give approximate equal weights to the sum of the inverse error terms in the parameters $\sigma_{0}\left(p_{\mathrm{i}}\right)^{2}$ and $\sigma_{\mathrm{P}}\left(p_{\mathrm{i}}\right)^{2}$ calculated from $\chi_{0}^{2}$ and $\chi_{\mathrm{P}}^{2}$ respectively. Neglecting the off diagonal terms results in $\sigma_{\mathrm{P}}\left(p_{\mathrm{i}}\right)^{2}=1 / \mathrm{A}_{0, \mathrm{ii}}$ and $\sigma_{\mathrm{P}}\left(p_{\mathrm{i}}\right)^{2}=1 / \mathrm{A}_{\mathrm{P}, \mathrm{i}}$; however to avoid numerical stabilities $K_{P}$ is written as shown in Eq. (2-6).

$$
\begin{equation*}
\mathrm{K}_{\mathrm{P}}=\sum_{\mathrm{k}}^{\mathrm{N}_{\mathrm{k}}} \operatorname{If}\left(\mathrm{Y}_{\mathrm{P}, \mathrm{kk}}\left\langle 10^{-14} \mathrm{~A}_{0, \text {,kk }}, 0,1.05 \mathrm{~A}_{0, \text { kk }} /\left(\mathrm{A}_{\mathrm{P}, \text { kk }}+\mathrm{A}_{0, \text { kk }} \operatorname{Min}\left(\mathrm{Y}_{\mathrm{P}, \mathrm{kk}} / \mathrm{Y}_{0, \text { kk }}, 0.05\right)\right)\right)\right. \tag{2-6}
\end{equation*}
$$

k corresponds to independent parameters that are a function of $\chi_{\mathrm{P}}^{2}$. Similarly for $\mathrm{K}_{\mathrm{R}}$ we have:

$$
\begin{equation*}
\mathrm{K}_{\mathrm{R}}=\sum_{\mathrm{k}}^{\mathrm{N}_{\mathrm{k}}} \operatorname{If}\left(\mathrm{Y}_{\mathrm{R}, \mathrm{kk}}\left\langle 10^{-14} \mathrm{~A}_{0, \mathrm{kk}}, 0,1.05 \mathrm{~A}_{0, \mathrm{kk}} /\left(\mathrm{A}_{\mathrm{R}, \mathrm{kk}}+\mathrm{A}_{0, \mathrm{kk}} \operatorname{Min}\left(\mathrm{Y}_{\mathrm{R}, \mathrm{kk}} / \mathrm{Y}_{0, \mathrm{kk}}, 0.05\right)\right)\right)\right. \tag{2-7}
\end{equation*}
$$

$\mathrm{K}_{\mathrm{R}}$ and $\mathrm{K}_{\mathrm{P}}$ can be modified using pen_weight and the macro Pen_Wt. Pen_Wt calls the macro Write_Pen_Wt which then has to be defined by the user. A definition that mimics the default is as follows:

```
macro Write_Pen_Wt(Aii, Ai, Pii, Pi)
{
    pen_weight = If(Pii<1e-14 Aii, 0, 1.05 Aii/(Pii + Aii Min(Pi/Ai, 0.05)));
}
```

$A_{i i}$ and $A_{i}$ corresponds to $A_{0, i i}$ and $Y_{0, i}$ respectively. For $K_{P}$ then $P_{i i}$ and $P_{i}$ corresponds to $A_{P, i i}$ and $Y_{P, i}$. For $K_{R}$ then $P_{i i}$ and $P_{i}$ corresponds to $A_{R, i i}$ and $Y_{P, i}$.

To formulate SheIX type restraints the following could be used:

```
pen_weight = 1;
pen\overline{alties_weighting_K1 = (Get(r_wp)/Get(r_exp))^2;}
do_errors_include_restraints
sa\overline{ve_best_chi2}
restrraint }\mp@subsup{}{}{-}=\operatorname{Sqrt(w) (yt-y);
```

where Sqrt(w) is simply the square root of the restraint weight used by SheIX.

### 2.7.1.....Improvements to Conjugate Gradient Solution method

The bound constrained conjugate gradient method (Coelho, 2005) used for solving the normal equations greatly assists in convergence of the non-linear least squares process. Previously min/max limits were calculated prior to the solution of the normal equations and then held constant during the solution process. Version 5 in addition dynamically recalculates min/max limits during the solution process for min/max limits that are a function of independent parameters. For example, to constrain site occupancies on three sites to full occupancy with three atomic species each with occupancy of 1 the following could be defined:

```
site Ni x .11 Y . 22 z . 33
    occ Ni nil 0.20000 min 0 max 1
    occ Zr zr1 0.30000 min 0 max = 1 - ni1;
    occ Ca cal = 1 - nil - zr1; : 0.50000
site Zr x . 21 Y . 32 z . 43
    Occ Ni ni2 0.40000 min 0 max = 1 - ni1;
    occ Zr zr2 0.50000 min 0 max = 1 - ni2;
    occ Ca ca2 = 1 - ni2 - zr2; : 0.10000
site Ca x . 31 y . 42 z . 53
    occ Ni ni3 = 1 - nil - ni2; : 0.40000
    occ Zr zr3 = 1 - zr1 - zr2; : 0.20000
    occ Ca ca3 = 1 - ca1 - ca2; : 0.40000
' Occupancy on sites add up to 1
prm = nil + zr1 + ca1; : 1.00000
prm = ni2 + zr2 + ca2; : 1.00000
prm = ni3 + zr3 + ca3; : 1.00000
' Individual species add up to 1
prm = nil + ni2 + ni3; : 1.00000
prm = zr1 + zr2 + zr3; : 1.00000
prm = ca1 + ca2 + ca3; : 1.00000
```

Version 4 allowed for such constraints but had difficulty in finding a minima without violating the limits.
Version 5 has no such difficulty as seen in test_exampleslocc-constrain.inp.

### 2.7.2.....Restraints and Penalties

A particular restraint can be reformulated into a penalty by squaring the restraint, for example:

```
restraint = a (x - b);
```

is equivalent to

```
penalty = a^2 (x - b)^2;
```

In the case of the restraint the off-diagonal terms $A_{R, i j}$ are calculated when approximate_ $A$ (the BFGS method) is not defined. In the case of the penalty the off-diagonal terms $A_{P, i j}$ is set to zero. Restraints often converge faster than equivalent penalties due to the use of the off-diagonal terms (compare ROSENBROCK10.INP with ROSENBROCK-10-RESTRAINT.INP). Penalties are useful for functions that are not to be squared; these include negative functions such as the GRS series atomic interaction (see ALVO4-GRSAUTO.INP).

For efficiency the $\mathbf{A}_{R}$ matrix is treated as a sparse matrix which is combined with $A_{0}$ (if it exists) where $A_{0}$ could be either sparse or dense. When approximate_A is used then the diagonal elements of $\mathbf{A}_{0}, \mathbf{A}_{P}$, and $\mathbf{A}_{\mathrm{R}}$ are not calculated; instead they are approximated by the BFGS method.

When approximate_ $A$ is used and both penalties and restraints are defined then this effectively means that the restraints are treated as penalties. The following for example:

```
Case 1
    approximate_A
    prm pl 1 prm rl 1
    penalty !P1 = 5^2 (p1 - 7)^2;
    penalty !P2 = 6^2 (p1 - 8)^2;
    restraint !R1 = 7 (r1 - 9);
    restraint !R2 = 8 (r1 - 10);
```

will have similar but not identical convergence to the following:

```
Case 2
    prm p1 1 prm r1 1
    penalty !P1 = 5^2 (p1 - 7)^2;
    penalty !P2 = 6^2 (p1 - 8)^2;
    penalty !P3 = 7^2 (r1 - 9)^2;
    penalty !P4 = 8^2 (r1 - 10)^2;
```

In Case 1 the diagonal elements of the A matrices are:

$$
\begin{aligned}
& A_{P, p 1 p 1}=(1 / 2) \partial^{2}(P 1+P 2) / \partial p 1^{2} \\
& A_{R, r 1 r 1}=(\partial R 1 / \partial r 1)^{2}+(\partial R 2 / \partial r 1)^{2}
\end{aligned}
$$

In Case 2 they are:

$$
\begin{aligned}
& A_{P, p 1 p 1}=(1 / 2) \partial^{2}(P 1+P 2) / \partial p 1^{2} \\
& A_{P, r 1 r 1}=(1 / 2) \partial^{2}\left(R 1^{2}+R 2^{2}\right) / \partial r 1^{2}
\end{aligned}
$$

The difference in behavior between penalties and restraints can be seen by comparing ROSENBROCK-
10.INP to ROSENBROCK-10-RESTRAINT.INP. In 500,000 iterations we have:

```
ROSENBROCK-10.INP: }71\mathrm{ iterations on average to convergence
ROSENBROCK-10-RESTRAINT.INP: 47 iterations on average to convergence
```

The restraints converge faster as the $A_{R, \mathrm{ij}}$ elements are calculated. Approximating $A_{R, \mathrm{j}}$ by defining approximate_ $A$ in ROSENBROCK-10-RESTRAINT.INP gives the fastest convergence time wise:

ROSENBROCK-10-RESTRAINT.INP: 71 iterations on average to convergence
Many penalties however cannot be formulated as a restraint, RASTRIGIN.INP for example, and in these cases penalties are mandatory.

### 2.7.3..... Saved refined values and save_best_chi2

Values saved on termination of refinement are determined as follows:

- If continue_after_convergence is NOT defined and save_best_chi2 is NOT defined then values saved corresponds to those of the last iteration.
- If continue_after_convergence is NOT defined and save_best_chi2 is defined then values saved corresponds to those that gave the best $\mathrm{Chi}^{2}$.
- If continue_after_convergence is defined and save_best_chi2 is NOT defined then values saved corresponds to those that gave the best Rwp.
- If continue_after_convergence is defined and save_best_chi2 is defined then values saved corresponds to those that gave the best Chi ${ }^{2}$

When there are no penalties or restraints then the best Chi ${ }^{2}$ corresponds to the best Rwp.

### 2.7.4..... Error calculation

Errors are calculated for all independent and non-independent parameters that are single valued when any of the following is defined:
do_errors: Errors calculated without the inclusion of penalties and restraints in the A matrix.
do_errors_include_penalties: Errors calculated with the inclusion of penalties in the A matrix.
do_errors_include_restraints: Errors calculated with the inclusion of restraints in the A matrix.

### 2.7.5.....Simulated annealing adaptive step size

The adaptive step size used in simulated annealing has been improved. In many case the complex temperature regime found in the macro Auto_T can be replaced with a single temperature. The example CIME-Z-AUTO.INP demonstrates the improvements by using a very incorrect starting temperature of 0.1 ; the program quickly modifies the temperature to a more appropriate vale. Output lines such as:

```
Breaking - randomize on errors revisit
```

indicate that a particular parameter configuration has been revisited and the temperature will be internally adjusted. Note, with randomize_on_errors, relative temperature values are pertinent and not absolute values.

### 2.7.6..... Refining on an arbitrary $\mathrm{Chi}^{2}$

The chi2 keyword allows for minimization of a user defined $\chi^{2}$. It can be a function of the reserved parameter names X, Yobs, Ycalc and SigmaYobs. In addition the keyword xdd_sum is a parameter that can be a function of these reserved parameter names. To, for example, define a normal least squares refinement the following can be used:

```
xdd...
    xdd_sum denominator = Yobs;
    xdd_sum numerator = (Yobs - Ycalc)^2 / Max(Yobs,1);
    chi2 = 100 Sqrt(numerator / denominator);
```

In refining on an arbitrary chi2 the first and second derivatives of chi2 with respect to each independent parameter is required. To do this fast Ycalc within chi2 is approximated with a first order Taylor approximation around the parameter vector $\mathbf{p}$. This approximation for various formulations of chi2 has yielded good convergence even for non-linear parameters. To summarize:

- chi2 is treated as a penalty
- For each independent parameter, a definite minima in chi2 is bracketed and inverse parabolic interpolation used to determine the minima of chi2 with respect to that parameter. In the calculation of chi2, Ycalc is replaced with its first order Taylor approximation and thus the full Ycalc is only calculated once per refinement iteration and not 100s of times.
- Finding the minima and the curvature of chi2 with respect to each parameter yields $1^{\text {st }}$ and $2^{\text {nd }}$ order derivatives of chi2 with respect to each parameter.
- The BFGS method (approximate_A) is then used to solve the resulting linear equations with off diagonal terms approximated according to the BFGS method.
- The BCCG method incorporating the Marquardt method with automatic Marquardt constant determination is used to solve the matrix equations.

The Rietveld refinement test_examplelchi2-ceo2.inp example demonstrates various scenarios.
Case 1) Here's output when NOT using chi2.

| 0 | Time | 0.05 | Rwp | 26.630 | 0.000 MC | 0.00 | 0 |
| ---: | :---: | ---: | :--- | ---: | ---: | ---: | ---: |
| 1 | Time | 0.06 | Rwp | 16.651 | -9.979 MC | 0.06 | 1 |
| 2 | Time | 0.06 | Rwp | 7.510 | -9.141 MC | 0.02 | 1 |
| 3 | Time | 0.08 | Rwp | 6.955 | -0.556 MC | 0.01 | 1 |
| 4 | Time | 0.08 | Rwp | 6.943 | -0.011 MC | 0.00 | 1 |
| 5 | Time | 0.08 | Rwp | 6.923 | -0.020 MC | 0.00 | 1 |
| 6 | Time | 0.09 | Rwp | 6.923 | -0.000 MC | 0.18 | 1 |
| -- | 0.094 | seconds | --- |  |  |  |  |

Case 2) Here's output when NOT using chi2 but using approximate_A.

| 0 | Time | 0.05 | Rwp | 26.630 | 0.000 MC | 0.00 | 0 |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| 1 | Time | 0.06 | Rwp | 16.883 | -9.747 MC | 0.00 | 0 |
| $\ldots$ |  |  |  |  |  |  |  |
| 16 | Time | 0.13 | Rwp | 6.950 | -0.002 MC | 0.04 | 1 |
| 17 | Time | 0.14 | Rwp | 6.949 | -0.002 MC | 0.09 | 1 |
| 18 | Time | 0.14 | Rwp | 6.949 | -0.000 MC | 0.29 | 1 |
| -- | 0.14 | seconds | --- |  |  |  |  |

Case 3) Here's output using chi2 defined for normal least squares

| 0 | Time | 0.03 | Rwp | 26.630 | 0.000 MC | 0.00 | 0 | P | 26.63020 |
| ---: | :--- | ---: | :--- | ---: | ---: | ---: | ---: | ---: | ---: |
| 1 | Time | 0.06 | Rwp | 15.897 | -10.733 MC | 0.00 | 0 | P | 15.89696 |
| $\ldots$ |  |  |  |  |  |  |  |  |  |
| 13 | Time | 0.33 | Rwp | 6.974 | -0.021 MC | 0.001 | P | 6.97366 |  |
| 14 | Time | 0.34 | Rwp | 6.958 | -0.016 MC | 0.001 P | 6.95755 |  |  |
| 15 | Time | 0.38 | Rwp | 6.951 | -0.006 MC | 0.001 P | 6.95122 |  |  |

The chi2 case (3) looks similar to case (2); however the path towards the minima is different as the chi2 procedure is very different to normal least squares refinement.

### 2.7.7..... Informing of unrefined parameters

Parameters that do not take part in a refinement are now reported, for example, the following:

```
prm a 1
prm b 1
```

where $a$ and $b$ are not used in any equations that are part of refinement will result in the following output:

```
Number of independent parameters not taking part in refinement: 2
    prm_10: a
    prm_10: b
```

The val_on_continue attribute of unrefined parameters are executed at the end of convergence. It can be useful, for example,

```
prm a 1 val on continue = b = 2; ' this sets the parameter b to 2.
```


### 2.8 Stacking faults

```
[site $name]...
    [layer $layer_name]
[stack $layer_name]...
    [sx E] [sy E] [sz E]
    [generate_these $sites]
        [generate_name_append $append_to_site_name]
```

The super cell approach to stacking faults has been implemented. layer identifies a site as belonging to a layer called \$layer_name. stack applies a stacking vector ( $s x, s y, s z$ ) to the named layer. Structures factors are generated in the usual manner; a shift corresponding to the stacking vector is then applied. stack operates in any space group. Sites that do not belong to a layer are treated as un-stacked and their structure factors are generated in the usual manner.
generate_these generates the sites found in \$sites for the stack with coordinates that reflect original \$sites positions plus the stacking vector. generate_name_append appends \$append_to_site_name to the generated site. The generated sites have occupancies set to zero which signals a dummy site. Dummy sites do not take part in structure factor calculations and hence speed is not hindered. The dummy sites allow for graphical display of the layers; ie.


Importantly penalties operate on dummy sites which allow restraints such as Distance_Restrain. For example,

```
space_group P1
site O1... layer A
site O2... layer A
stack A
    SX...
    generate_these O1
        generāte_name_append __1
append_fractional
    in_s_st_format
```

will output for append_fractional the following:

```
site O1 ...
site O2...
site O1_1 ... Occ O 0
```

The test_examples\stacking-faults\kaolinite.inp shows how to simplify the setting up of layers with the use of simple macros. Speed of calculation for structure factors are very fast and the derivatives of the stacking vectors $\{s x, s y, s z\}$ are very fast. The main bottle neck in speed is summing the peaks to Ycalc. The switch "\#define Speed" in kaolinite.inp shows keywords that can speed things up in the early stages of determining the stacking vectors.

### 2.8.1..... Fitting to a Debye-formulae generated pattern using 'stack'

A test pattern was generated using the Debye scattering equation. The structure comprised a single atom in an Orthorhombic unit cell with 40 layers ( $40 \times 40 \times 40$ unit cells) in the a-b plane shifted according to $\{\operatorname{Round}(\operatorname{Rand}(0,2)) / 3, \operatorname{Round}(\operatorname{Rand}(0,2)) / 3,0\}$. The blue line in the following is the generated pattern comprising the average of 30 runs of the Debye scattering equation. The red line corresponds to a Rietveld fit of 6 super cell structures $(1 \times 1 \times 40)$ showing that the super cell approach is a good approximation to the Debye formulae for this example.


The example stacking-faults\debye-new.inp corresponds to the Rietveld fit using the layer and stack keywords. The debye-old.inp file corresponds to the same Rietveld fit but without the layer and stack keywords; instead layers are explicitly defined using site in an enlarged unit cell.

There are two time consuming bottle necks dealt with:

1) Summing peaks to Ycalc
2) Calculating structure factors for the stacked layers

The new phase dependent keyword called [del_approx \#] groups peaks from the peaks buffer whilst summing peaks to Ycalc; the peaks are grouped such that their 2Th positions all lie within:
-del_approx Peak_Calculation_Step < 2Th < del_approx Peak_Calculation_Step

Once the group is found then only the two peaks with the smallest and largest 2Th is kept. The in-between peaks have their intensities appropriated to the kept peaks. A particular I(2Th) intensity has its intensity distributed to the two end peaks as follows:

```
I (2Th_
I (2Th 2) = I (2T) (1 - f)
```

where,


The peak buffer stretching routines have also been optimized for both accuracy and speed. The following points should be noted when working with large super cells

- The layer and stack keywords increase computational speed and reduce memory usage
- del_approx increase computation speed at a relatively small cost to accuracy; a value between 1 and 3, dependent on Peak_Calculation_Step, is typically acceptable.
- The graphical display of 10 s of 1000 s of hkl ticks (there's 51584 hkls in each phase of the debyenew.inp) is time consuming; turning the graphical hkl ticks option Off is worthwhile.


### 2.8.2..... Fitting to Kaolinite data

stacking-faults $\backslash$ kaolinite.inp demonstrates the application of stack and layer with the following fit:


In this example the stacking vectors are refined in a simulated annealing process.

### 2.9 Laue refinement

Single crystal Laue diffraction data can be refined; data files have the extension *.hkl-lam; see directory test_exampleslaue. Laue_Lam is a reserved parameter name that can be used in hkl type equations; it returns the reflection dependent wavelength. The merging of equivalent reflections and Friedel_pairs are not allowed with Laue refinement; the following keywords are internally defined with Laue refinement:

```
dont_merge_equivalent_reflections
dont_merge_Friedel_pairs
```

and the following messages reported:

```
Equivalent reflections not merged
Friedel pairs not merged
```


### 2.10 Quantitative Analysis

The following are items associated with quantitative analysis with new items in red:

```
xdd..
    mixture_MAC
    mixture-density g on cm3
    weight \overline{percent amōrp\overline{hous}}\mathbf{}\mathrm{ - }
    elemental composition
    element_wēight_percent...
    element_weight_percent_known...
    prm = Get(sum_smvs)...
    Mixture_LAC_1_on_cm(0) ' macro
    str...
        weight_percent
        cell_mass
        cell-volume
        phase_MAC
        spike\overline{d phase measured weight percent}
        correc\overline{ted_weight_percēnt}
        prm = Get(sum_smvs)...
        prm = Get(smv)
        prm = Get(sum_smvs_minus_this)...
        prm = Get Ele\overline{ment \overline{W}}\mathrm{ ight(atom)...}
        Phase_LAC_1_on_cm(0) ' macro
        Phase_Densi\overline{t}y_\overline{g}_on_cm3(0) ' macro
```

Test_exampleslquantlquant-1.inp uses many of these and additionally writes equivalent terms in the form of equations, for example:

```
' This is weight percent
prm = 100 Get(smv}) / Get(sum_smvs); : 0
prm q = spiked phase measured weight percent /
    spiked_phase__measurēd_weigh̄t_percent_wt; : 0
    ' This is corrected_weight_percent
prm = q Get(weight_percent); : 0
    ' This is weight_percent_amorphous
    prm = 100 (1 - q); ; 0
```


### 2.10.1...Elemental weight percent constraint

The QUANT implementation is almost entirely written internally using the TOPAS Symbolic system. Dependencies are automatically taken care of and unnecessary recalculations kept to a minimum. The overriding plus however is the flexibility it allows. If for example an elemental weight percent was known and three phases of the mixture comprised this element then Get_Element_Weight can be used to get the weight of the element as a function of the structure; ie.

```
str...
    prm z1 = Get_Element_Weight(Zr);
    MVW(!m1 0, !v1 0,0)
str...
    scale s2 0.001
    prm z2 = Get_Element_Weight(Zr);
    MVW (0, !v2 0,0)
str...
    scale s3 0.001
    prm z3 = Get_Element_Weight(Zr);
    MVW(0, !v3 0,0)
```

Rearranging the formulae for element weight percent, the scale parameter of one of the phases, say the first one, can be written as follows:

```
scale = (0.01 known_Zr Get(sum_smvs_minus_this) - s2 v2 z2 - s3 v3 z3) / (v1
    (z1 - 0.01 known_Zr ml));
```

Get(sum_smvs_minus_this) returns the sum of SMVs minus the phase where it is defined. The test_examplelquantlquant-3.inp demonstrates this constraint with very good convergence. It comprises 4 phases with three of them comprising Zr atoms. Quant-2.inp demonstrates constraining a weight percent to a known value using the macro:

```
macro Known_Weight_Percent(& w)
    {
        scale = (w/(100-w)) Get(sum_smvs_minus_this) / (Get(cell_mass)
            Get(cell_volume));
    }
```


### 2.10.2...Elemental composition and Restraints

The xdd dependent keyword element_composition reports the elemental composition for atoms within the structures of the $x d d$, for example:

## Before Refinement:

```
xdd...
    elemental_composition
```

After Refinement:

```
xdd...
    elemental_composition
    {
\begin{tabular}{lr} 
& Rietveld \\
AL & \(0.875:-0.021\) \\
O & \(26.135:-0.009\) \\
SI & \(0.090:-0.003\) \\
YR & \(6.289:-0.012\) \\
Y & \(66.612:-0.029\)
\end{tabular}
```

element_weight_percent \$ELEMENT \$NAME \#: is an xdd dependent keyword that returns the weight percent of an element within the corresponding str's of the $x d d$. Example usage:

Before Refinement:

```
penalties_weighting_K1 .1
xdd...
    element weight percent Zr+4 zr 0
    restraint = (z\overline{r}-65); : 0
```

After Refinement:

```
penalties_weighting_K1 .1
xdd...
    element weight_percent Zr+4 zr 65.0275252`
    restrain}t=(z\overline{r}-65); : 0.0275251892
```

In this example zr is the name given to the element $\mathrm{Zr}+4$ and the restraint shows a known value of 65 (set for example by XRF results). The refinement obeys the restraint according to the value set for
penalties_weighting_K1.
For restraining a weight percent the following can be used:

```
xdd...
    penalties_weighting_K1 .2
    restraint = (Cubic_Zirconia_wt_percent - 36); : 0
    str...
        MVW(0,0, !Cubic_Zirconia_wt_percent 0)
```

Note, the Cubic_Zirconia_wt_percent name which is given to the weight percent; see test examples\quant.

### 2.10.3... Amorphous phase composition

If spiked_phase_measured_weight_percent is defined then elemental_composition will report on Rietveld values, Corrected values as well as values from the original un-spiked sample. If element_weight_percent_known keywords are defined then elemental_composition will additionally report on the elemental contents of the amorphous phase, for example, from test_examples\quantlquant-1.inp we have:

|  | Rietveld | Corrected | Original | Other |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| AL | $1.176^{{fcc185910-1a14-4a05-ad24-4726c505ea63} 0.000$ |  |  |  |  |  |
| 0 | $26.271{ }^{\text {- }} 0.017$ | $23.640{ }^{-} 0.832$ | $23.162 {f77ea015b-505b-4b41-ba60-44842bc92c83}{ }^{-} 0.013$ | 5.563 ${ }^{-} 0.204$ | 5.676 ${ }^{\text {- }} 0.209$ | $0.000{ }^{-} 0.000$ |
| ZR | $66.267^{-}{ }^{-} 0.055$ | $59.631{ }^{-}{ }^{-} 2.185$ | $60.847^{-}-2.229$ | $2.153{ }^{-}{ }^{-} 2.229$ |  |  |
| Other | $0.000{ }^{-}$-0.000 | $10.015{ }^{\text {¢ }}$-3.224 | $10.219{ }^{\text {- }}$ - 3.290 | $7.228{ }^{-}$-0.212 |  |  |

The first second and third columns sum to $100 \%$. The second column corresponds to corrected values including the spiked phase. The third and fourth columns relate to elemental weight percents of the original phase. The last row of the second column (in purple) corresponds to Get(weight_percent_amorphous), the last row of the fourth column (in red) is the amount that is undefined; it comprises the Green number minus the elements of the third column excluding the last row. Note the zeros for Al (in blue); this is due to the spiked phase (dummy test data) being the only phase containing Al.

### 2.10.4... Using a dummy_str phase to describe amorphous content

If it is known that the amorphous content, (purple number) in the above table comprises a known composition, say $\mathrm{TiO}_{2}$, then a dummy_str can be used to describe the amorphous content as follows:

```
dummy_str
    phäse_name "Amorphous"
    a 5 b 5 c 5
    space_group 1
    site Ti occ Ti 1
    site O occ O 2
    Known_Weight_Percent(10.0148)
    MVW(0, 0 ,0)
```

*** Note: dummy_str's void of MVW and sites takes no part in Quantitative analysis.
The lattice parameters and the chemistry should correspond to a real structure in order for Mixture_LAC_1_on_cm and phase_LAC to be correctly calculated; in the case of using the Brindley correction these changed values will change the quantitative results. The space group entry can be other than P1 so long as the chemistry is correct. Inclusion of the dummy_str produces:

```
elemental_composition
{
                                    Rietveld
    AL 1.059` 0.038
    O 27.652`-0.015
    SI 0.094`-0.003
    0.094-0.003
    5.563`-0.012
    59.631`-0.050
    0.000`-0.000
\begin{tabular}{c} 
Corrected \\
1.059 \\
\(27.652-0.000\) \\
\(0.094-0.075\) \\
6.002 \\
\(5.563-0.215\) \\
59.631 \\
0.000 .185 \\
\hline
\end{tabular}
Original
0.000
\(27.256-0.000\)
0.096
\(6.125-0.005\)
5.676
\(60.847-0.219\)
0.000
```

Note that the 'Other' row becomes zero as the dummy_str has been assigned the amorphous content. The change in mixture values are:

## Without dummy_str

```
Mixture_LAC 1 on_cm( 557.47740`0.58665)
mixture_density_g_on_cm3 5.26713308`_0.00292681843
```


## With dummy_str

```
Mixture_LAC_1_on_cm( 608.85143` 0.76954)
mixture_density_\overline{q_on_cm3 5.866\overline{01008`_0.00407998952}}\mathbf{\}=0
```

If XRF results were entered for element_weight_percent_known, for example:

```
element_weight_percent_known Zr 63
element_weight_percent_known O 24
```

Then we get:

```
elemental_composition
```

\{

|  | Rietveld | Corrected | Original | Other |
| :---: | :---: | :---: | :---: | :---: |
| AL | $1.059{ }^{\text { }}$-0.038 | $1.059{ }^{\text {- }} 0.000$ | $0.000 \times 0.000$ | $0.000 \times 0.000$ |
| $\bigcirc$ | $27.652{ }^{-} 0.015$ | $27.652{ }^{-} 0.975$ | 27.256 ${ }^{-} 0.995$ | -3.256 ${ }^{-} 0.995$ |
| SI | $0.094{ }^{-}{ }^{-0.003}$ | $0.094{ }^{-}+0.005$ | $0.096^{-}-0.005$ | $0.000{ }^{-} 0.000$ |
| TI | $6.002{ }^{-} 0.000$ | $6.002^{-} 0.215$ | $6.125^{-} 0.219$ | $0.000{ }^{-} 0.000$ |
| Y | $5.563{ }^{-}{ }^{-} 0.012$ | $5.563{ }^{\text {- }} 0.204$ | 5.676 ${ }^{-}$-0.209 | $0.000{ }^{-} 0.000$ |
| ZR | $59.631{ }^{-} 0.050$ | $59.631{ }^{-} 2.185$ | $60.847^{-} 2.229$ | $2.153^{-} 2.229$ |
| Other | $0.000{ }^{-}{ }^{-0.000}$ | $0.000{ }^{\text {- }} 3.583$ | $0.000{ }^{\text {- }} 3.656$ | $1.103{ }^{\text {- }}$-0.431 |

The negative element weight percent for $O$ for the amorphous content reflects the fact that the measured XRF value for O is lower than the refinement's value (note, this example is simply for testing and the XRF values used are fictitious).

### 2.10.5... Quant using hkl_ls or other non-str phases

dummy_str's can be used to represent the quantitative results arising from non-str phases. For example, consider a phase where the structure is unknown but the chemistry is known and a calibration constant has been determined relating the hkl_Is intensities to the scale parameter of the $h k l \_I s$ phase. In such a case the dummy_str can be written as (see Quant-6.inp):

```
dummy_str
    phäse name "Linked Cubic Zirconia"
    Cubic(5.137866)
    space_group F_M_-3_M
    site \overline{Zr x 0}-\quad y 0 z 0 occ Zr 0.85
    site O x 0.25 y 0.25 z 0.25 occ 0 0.962
    scale = hkl_scale;
    Phase LAC 1-on cm(0)
    Phase_Density_g_on_cm3(0)
    MVW (0- 0 ,0)
```

Note in this case a space group has been entered with structural parameters that looks like a known structure; this could occur for example where the structure is known in an ordered state but the diffraction pattern comprises a disordered state. In other cases the P1 space group may suffice with site occupancies corresponding to the appropriate chemistry. The dummy_str is linked to the $\underline{h k l \_I s}$ phase by assigning it scale parameter to the hkl_Is scale parameter. Quant-7.inp is a similar process except that a fit_obj is linked to a dummy_str. Graphically the linked dummy_str is plotted with the calculated pattern of the $h k l \_I s$ phase or fit_obj, for example, Quant-7.inp produces:


Here the Blue line corresponds to the dummy_str which plots the calculated pattern of the linked fit_obj which in turn comprises a user_y object. The weight percent value determined by the dummy_str is also displayed.

### 2.10.6...Summary of Quant examples

- Quant-1.inp: a general example showing the use of element_weight_percent_known etc...
- Quant-2.inp: uses the Known_Weight_Percent macro
- Quant-3.inp: uses elemental constrain using Get_Element_Weight
- Quant-4.inp: uses an hkl Is phase instead of a str phase; uses Known_Weight_Percent on the hkl_ls phase.
- Quant-5.inp: uses a dummy_str to describe an amorphous phase
- Quant-6.inp: uses a hkl_ls phase to describe a phase; links a dummy_str to the hkl_ls phase to get QUANT info.
- Quant-7.inp: uses a fit_obj that is a function of a user y object to describe a phase; links a dummy_str the fit_obj to get QUANT info.


### 2.10.7...External standard method

The method of O'Connor and Raven (1988) has been implemented in both GUI and Launch modes through the use of the macros:

```
macro K_Factor_MAC_K(mac, k, tot)
{
    move to xdd
    loca\overline{l !k_factor_mac_local mac}
    local !k_factor_k_lōcal_ \overline{k}
    local !k_factor_sum_wps_= = 0; : tot
}
macro K_Factor_WP(result)
{
    local k_factor_wp_ = 1.6605402 Get(smv) k_factor_mac_local_ /
        k_factor_k_\`ocal_; : result
    if P\overline{rm_There}(\overline{k}_factoror_sum_wps_) {
        existing_prm k_factor__sum_wps_ += k_factor_wp_;
    }
}
```

See test_examples\k-factor.

### 2.11 Learnt Shapes for Background or Otherwise

The new keywords user_y and fo_transform_ $X$ provides a means to use learnt shapes as a background function. The test example USER_Y.INP produces the following fit to Quartz using a learnt Pseudo-Voigt.


Example usage:

```
user_y NAME { #include SOME_FILE }
user_y NAME SOME_FILE
fit obj = NAME;
    fo_transform_X = (X - x) / s;
```

The user defined NAME corresponds to a parameter name given to the user_y; it can be used in all equations that can be a function of $X$, for example:

```
fit_obj = Exp(NAME^2);
```

fo_transform_ $X$ is a dependent of fit_obj and it transforms the $X$ used within the fit_obj. For example, NAME could have an $x$-axis that does not match the $x$-axis of the Yobs pattern; fo_transform_ $X$ provides a means to transform the Yobs $x$-axis to the user_y $x$-axis.

The user_y NAME $\{\ldots\}$ usage allow shapes to be typed directly into the INP file using the _x1_dx tag. A triangle for example is formulated as follows :

```
user_y NAME
    _x1_dx -1 1/* the start x and step */
    0 1-0 /* the shape data */
    }
```

More than one user_y can be defined and they can be used any number of times in equations that can be a function of $X$. The test example USER_Y.INP loads a single shape, and stretches and scales it five different ways onto a diffraction pattern to fit the Quartz triplet. Convergence is as fast as with any other other refinement.

### 2.12 Emission Profile with Absorption Edges

```
lam...
    [modify_peak]
        [modify_peak_apply_before_convolutions]
        [modify_peak_eqn !E] : Can be a function of Get(current_peak) and
                                Get(current_peak_x)
            [current_peak_min_x !E]
            [current_peak_max_x !E]
```

The modify_peak keyword can be used to modify peak profiles either before convolutions or after; here's a plot from AL2O3-Spinnel-PAM.INP (see directory test_exampleslabsorption-edge) that has an identical absorption edge modelled for both $\mathrm{Al2O} 3$ and Spinnel samples:

modify_peak functionality is realized by using the internal data objects of Get(current_peak_x) and Get(current_peak). These two objects return the x-axis wavelength being worked on by the program and the current calculated peak intensity at that $x$-axis position respectively.

### 2.13 scale_phase_X keyword

The scale_phase_X keyword scales Ycalc point by point. It can be used to define say alternate Lorentz Polarization factors. Some main points:

- Can be a function of $X$
- Multiple definitions are allowed and each is applied to the pattern.
- Can occur at the $x d d$ or phase level.

Here's an example:

```
xdd...
    scale_phase_X...
    str...
        scale_phase_X...
    hkl Is...
        scale_phase_X...
```

The first str is multiplied by the first and second scale_phase_ $X$; the hkl_Is phase is multiplied by the first and third scale_phase_ $X$.

### 2.14 Magnetic Structure Refinement

```
str...
    [mag_only_for_mag_sites]
    [mag_space__group $symbol]
    site...
        [mlx E] [mly E] [mlz E] [mg E]
        [mag_only]
        ' site dependent macros
        MM_CrystalAxis_Display(mxc, myc, mzc)
        MM_CrystalAxis_Refine(mxc, mxv, myc, myv, mzc, mzv, mlx_v, mly_v, mlz_v)
        MM-Cartesian Display(mxc, myc, mzc)
        MM_Cartesian_Refine(mxc, mxv, myc, myv, mzc, mzv, mlx_v, mly_v, mlz_v)
```

Thanks to Branton Campbell and John Evans for expert assistance during the implementation of magnetic refinement. Magnetic refinement is implemented using the keywords $m / x, m / y, m / z, m g$ and
mag_space_group. See example in the test_examples\mag directory as well as the tutorial by John Evans at http://www.dur.ac.uk/john.evans/topas_workshop/tutorial_lamno3_magnetic.htm.

The Magnetic intensity is given by:
Magnetic intensity = Fmagcperp . Fmagcperp ${ }^{*}=\mid$ Fmagcperp $\mid$
where the superscript * denotes conjugate gradient and:
Fmagcperp $=$ Fmagc $-($ Fmagc. Qhat $)$ Qhat

Or in words, Fmagcperp is the component of the magnetic vector in the direction perpendicular to the scattering vector $\mathbf{Q}$, where
$\boldsymbol{Q}=\left(\mathbf{L}^{-1}\right)^{\top}{ }^{*} \mathbf{h}$
$\mathbf{Q h a t}=\mathbf{Q} /|\mathbf{Q}|$
where
$\mathbf{L}$ is the Cartesian lattice parameters in $3 \times 3$ matrix form
$\mathbf{h}$ is the Miller indices in vector form

* denotes matrix multiplication

Superscript ${ }^{-1}$ denotes matrix inverse
Superscript ${ }^{\top}$ denotes matrix transpose
$\left(\mathbf{L}^{-1}\right)^{\top}=$ reciprocal lattice parameters
Fmagc in terms of the Cartesian lattice parameters is:
Fmagc $=\mathbf{L}$ * Fmag

Fmag for the plane $\mathbf{h}$ for a single site is:
Fmag $=\sum_{j}\left(\mathbf{B}_{\mathrm{j}}{ }^{*} \mathbf{m}\right) \operatorname{Exp}\left(2 \pi \mathrm{i} \mathrm{U}_{\mathrm{j}}\right)$
where the summation is over the equivalent positions $j$ and
$\mathrm{U}_{\mathrm{j}}=\mathbf{h} . \mathbf{R}_{\mathrm{j}} \mathbf{x}+\mathbf{h} . \mathbf{t}_{\mathrm{j}}$
$\mathbf{x}=\{x, y, z\}=$ site fractional coordinates
$\mathbf{m}=\{\mathrm{mlx}, \mathrm{mly}, \mathrm{mlz}\}=$ magnetic moment
$\mathbf{R}_{j}=$ rotation part of space group operator
$\mathbf{t}_{j}=$ translational part of space group operator
$\mathrm{d}_{\mathrm{j}}=\mathrm{s}_{\mathrm{j}} \operatorname{determinant}\left(\mathbf{R}_{\mathrm{j}}\right)=\mathrm{s}_{\mathrm{j}} \operatorname{det}\left(\mathbf{R}_{\mathrm{j}}\right)$
$\mathbf{B}_{\mathrm{j}}=\mathrm{s}_{\mathrm{j}} \operatorname{det}\left(\mathbf{R}_{\mathrm{j}}\right) \mathbf{R}_{\mathrm{j}}=$ magnetic transformation matrix
The file MAGDATA.DAT (a GSAS file - permission for its use granted from Robert Von Dreele, the author of GSAS) comprises data for calculating magnetic form factors. The Lande splitting factor can be refined using the site dependent parameter $m g$; defaults for $m g$ are obtained from MAGDATA.DAT. Shubnikov groups are obtained from the file SHUBNIKOVGROUPS.TXT.
mag_only: When defined the x-ray component to intensity for the site in question is ignored.
mag_only_for_mag_sites: When defined the x-ray component to intensity for all magnetic sites for the str in question is ignored.

### 2.14.1... Magnetic refinement warnings/exceptions

The following two messages:

1) Warning: Magnetic moment mlx of site Fe has no contribution to Fmag
2) Magnetic moment mlx of site Fe cannot be refined as it has no derivative
arise when for each group of equivalent positions of a special position the first row of the matrix $\sum_{j} \mathbf{B}_{j}{ }^{*} \mathbf{m}$ is zero where the j's sum over the equivalent positions of a special position group. Similar messages for mly and $m / z$ are given. Note, the fact that $m / x, m l y, m / z$ may or may not be refined and their associated constraints are considered. Refinement terminates in the case of message (2) when $m l x$ is being refined.

### 2.14.2...Displaying Magnetic moments

Magnetic moments (Occupancy $\mathbf{B}_{j}{ }^{*} \mathbf{m}$ ) are displayed graphically when 'view_structure' is defined. For the case where the atom balls are masking the display of the magnetic moment arrows the "Atom size" can be varied as shown in the following:


### 2.14.3... 'Decomposing' Fmag for speed

When using magnetic space groups other than 1.1 equivalent positions of the space group are written in terms of other equivalent positions.

Let
$\mathrm{C}_{\mathrm{j}}=\cos \left(\mathrm{U}_{\mathrm{j}}\right)$
$S_{j}=\sin \left(U_{j}\right)$
$\operatorname{Exp}(\mathrm{i} U)=\mathrm{C}_{\mathrm{j}}+\mathrm{i} \mathrm{S}_{\mathrm{j}}=$ Euler's formulae
For two equivalent positions of a special position we have
$\mathrm{U}_{1}=\mathrm{U}_{2}=\mathrm{U}$
$\mathbf{F m a g}_{1}+\mathbf{F m a g} \mathbf{m}_{2}=\mathbf{s}_{1} \operatorname{det}\left(\mathbf{R}_{1}\right) \quad \mathbf{R}_{1} \mathbf{m} \operatorname{Exp}(\mathrm{i} \mathrm{U})+\mathbf{s}_{2} \operatorname{det}\left(\mathbf{R}_{2}\right) \quad \mathbf{R}_{2} \mathbf{m} \operatorname{Exp}(\mathrm{i} \mathrm{U})$

$$
\begin{aligned}
& =\left(\mathrm{s}_{1} \operatorname{det}\left(\mathbf{R}_{1}\right) \mathbf{R}_{1}+\mathrm{s}_{2} \operatorname{det}\left(\mathbf{R}_{2}\right) \mathbf{R}_{2}\right) \mathbf{m} \operatorname{Exp}(\mathrm{i} \mathrm{U}) \\
& =\mathbf{c} \mathbf{m} \operatorname{Exp}(\mathrm{i} \mathrm{U})
\end{aligned}
$$

$\mathbf{c}$ is independent of $\mathbf{x}$
Note, a particular special position could have many equivalent positions.
If $\mathbf{R}_{1}=-\mathbf{R}_{2}$ and $\mathbf{t}_{1} \equiv-\mathbf{t}_{2}$ for two equivalent positions then

$$
\mathrm{U}_{1}=-\mathrm{U}_{2}=\mathrm{U}
$$

$\mathbf{F m a g}_{1}+\mathbf{F m a g}_{2}=\mathrm{s}_{1} \operatorname{det}(\mathbf{R}) \mathbf{R} \mathbf{m} \operatorname{Exp}(\mathrm{i} \mathbf{U})+\mathrm{s}_{2} \operatorname{det}(-\mathbf{R})(-\mathbf{R}) \mathbf{m} \operatorname{Exp}(-\mathrm{i} \mathbf{U})$
Now,
$\operatorname{det}(\mathbf{R}) \mathbf{R}=\operatorname{det}(-\mathbf{R})(-\mathbf{R})$
or,
$\mathbf{F m a g}_{1}+\mathbf{F m a g}_{2}=\operatorname{det}(\mathbf{R}) \mathbf{R} \mathbf{m}\left(\mathrm{s}_{1} \operatorname{Exp}(\mathrm{i} \mathbf{U})+\mathrm{s}_{2} \operatorname{Exp}(-\mathrm{i} \mathbf{U})\right)$
For $\mathrm{s}_{1}=\mathrm{s}_{2}$
$\mathbf{F m a g}_{1}+\mathrm{Fmag}_{2}=\mathrm{s}_{1} \operatorname{det}(\mathbf{R}) \mathbf{R} \mathbf{m} 2 \mathrm{C}$
For $\mathrm{s}_{1}=-\mathrm{s}_{2}$
$\mathbf{F m a g}_{1}+\mathbf{F m a g}_{2}=\mathrm{s}_{1} \operatorname{det}(\mathbf{R}) \mathbf{R} \mathbf{m}(2 \mathrm{i} \mathrm{S})$

## If $\mathbf{R}_{1}=\mathbf{R}_{2}$ for two equivalent positions then

```
\(\mathbf{F m a g}_{1}+\mathbf{F m a g}_{2}=\mathbf{s}_{1} \operatorname{det}(\mathbf{R}) \mathbf{R} \mathbf{m} \operatorname{Exp}(\mathbf{i} \mathbf{h} . \mathbf{R} \mathbf{x}) \operatorname{Exp}\left(\mathrm{i} \mathbf{h} . \mathbf{t}_{\mathbf{1}}\right)+\mathrm{s}_{\mathbf{2}} \operatorname{det}(\mathbf{R}) \mathbf{R} \mathbf{m} \operatorname{Exp}(\mathbf{i} \mathbf{h} . \mathbf{R} \mathbf{x}) \operatorname{Exp}\left(\mathrm{i} \mathbf{h} . \mathbf{t}_{\mathbf{2}}\right)\)
    \(=\operatorname{det}(\mathbf{R}) \mathbf{R} \mathbf{m}\left(\mathbf{s}_{1} \operatorname{Exp}\left(\mathbf{i} \mathbf{h} \mathbf{t}_{1}\right)+\mathbf{s}_{2} \operatorname{Exp}\left(\mathbf{i} \mathbf{h} \cdot \mathbf{t}_{2}\right)\right) \operatorname{Exp}(\mathbf{i} \mathbf{h} . \mathbf{R} \mathbf{x})\)
    \(=\mathbf{c} \operatorname{Exp}(\mathrm{i} \mathbf{h} . \mathbf{R} \mathbf{x})\)
```

$\mathbf{c}$ is independent of $\mathbf{x}$ and is calculated only once.
Many R's can be the same for a particular space group with only the t's changing.

## Calculating C and S

$\operatorname{Exp}(\mathbf{i}(\mathbf{h} . \mathbf{R x}+\mathbf{h} . \mathbf{t}))=\operatorname{Exp}(\mathbf{i} \mathbf{h} \mathbf{R} \mathbf{x}) \operatorname{Exp}(\mathrm{i} \mathbf{h} . \mathbf{t})$
$\operatorname{Exp}(\mathbf{h} . \mathbf{t})$ is constant for a particular $\mathbf{h}$ and is calculated only once.
Only unique Exp(i h.Rx) are calculated.
Trigonometric recurrence is used to calculate sines and cosines resulting in three cosine and three sine operations per unique equivalent $r$. In other words a sin and cos is not calculated for each $h$.
Note a sin or cos function is equivalent to about 40 to 60 multiplies.

### 2.15 Refining on $\mathbf{f 0}, f^{\prime}$ and $f^{\prime \prime}$

```
[f0_f1_f11_atom]...
    [f0 E] [f1 E] [f11 E]
```

Example usage is as follows:

```
report_on_str
load f\overline{0}_f\overline{1}_f11_atom f1 f11 {
    Ba @--0.160\overline{1}27754 2.3954287
    Ge 0.184162081 1.86162161
}
```

High correlations exist between $f 1$ and $f 11$, scale and beq parameters.
The f0_f1_f11_atom keyword can be used at the str, $x d d$ and global levels. f' or f' can be defined and refined independently. Defaults are used when either $f^{\prime}$ ' or $f^{\prime \prime}$ are not defined. The XRAY-POWDER.INP and TOF.INP in the directory test_examples $1 \mathrm{f0} 0 \mathrm{f} 1-\mathrm{f} 11 \backslash$ demonstrates the use of $f 0, f 1$ and $f 11$.

The $f 0$ parameter can be a function of the reserved parameter D_spacing; for example:

```
prm al 25 min -50 max }5
load f0_f1_f11_atom f0 f11 {
```

```
        Pb+2
        = a1 Exp(1.058874 (-0.25) / D_spacing^2) +
        16.496822 Exp(0.106305 (-0.25) / D_spacing^2) +
        19.984501 Exp(6.708123 (-0.25) / D_spacing^2) +
        6.813923 Exp(24.395554 (-0.25) / D_spacing^2) +
        5.233910 Exp(1.058874 (-0.25) / D_spacing^2) +
        4.065623; ' this is f0 for Pb
        @ 5 ' this is f11 for Pb
}
```

For X-ray data $f 0$ is by default obtained from the file atmscat.cpp. For neutron data the $f 0$ parameter corresponds to the neutron scattering length. Defaults for neutron scattering lengths are obtained from the file neutscat.cpp. Neutron scattering lengths can be refined as demonstrated in test_exampleslf0-f1f11\TOF.INP.

- Keyword no_f11 instructs the program to ignore f11. This increases speed with little change in Ycalc.
- Keyword report_on_str reports on $f 1$ and $f 11$ or neutron scattering lengths used. No values are reported when the keyword d_spacing_to_energy_in_eV_for_f1_f11 is used.

To disable the effects of $f 0, f 1$ and $f 11$, for say CeO 2 , then the following could be used:

```
load f0_f1_f11_atom f0 f1 f11 {
    Ce+4 1 0 0
    0-2 1 0 0
}
```


### 2.15.1 ... Invalid f1 and f11

The following message is displayed when there are no valid entries for $f^{\prime}$ and $f^{\prime \prime}$ in the corresponding NFF file:

```
Invalid f1 and f11 for O in file ...\ssf\o.nff
for the wavelength 0.399826.
Setting value(s) to zero
```

In such cases the user may choose to manually define f' and f" using $f 1$ and $f 11$ keywords respectively, see test_examples\f0-f1-f11 directory. Also of use is to view f' and f" NFF files found in the SSF directory using the GUI Tool menu; ie.


### 2.16 Isotopes and Atom Names

The file macltab1.html is no longer used. Instead isotopes.txt is used for obtaining isotope weights. It's now possible to have the following when refining either neutron (ie. the keyword neutron_data is defined) or x-ray data and to obtain the correct results without changing the INP str:

```
site ... occ Mg ...
site ... occ Mg+2 ...
site ... occ 24Mg ...
site ... occ 26Mg ...
site ... occ 26Mg+2 ...
```

In the cases of ' $M g$ ' and ' $M g+2$ ' the atomic weight used is the "Standard Weight" as defined in isotopes.txt.

In the cases of ' 26 Mg ' and ' $26 \mathrm{Mg}+2$ ' the atomic weight used is the isotope weight as defined in isotopes.txt. Note the ' +2 ' is dropped when searching that file.

The atomic weight for 24 Mg is not the same as that for Mg . When 24 Mg is used then the isotope weight for 24 Mg is used. When Mg is defined then the Standard weight is used. The Standard weight corresponds to the mean weight of the naturally occurring Mg isotopes.

In the case of $x$-rays:

- atomic scattering factors used (from file atmscat.cpp) for 26 Mg and $26 \mathrm{Mg}+2$ corresponds to those of Mg or $\mathrm{Mg}+2$ respectively. The numbers occurring at the start of the symbol is dropped when searching atmscat.cpp.
- f' and f' corrections (files in ssf directory) corresponds to that for Mg. In other words the numbers occurring at the start of the symbol as well as the charge (ie. ' +2 ' in this case) is dropped.

In the case of neutrons:

- scattering lengths used are from the neutscat.cpp file; the charge (ie. '+2') is dropped when searching neutscat.cpp.

Internally the program converts ' D ' and ' T ' to ' 2 H ' and ' 3 H ' respectively.

### 2.17 An Accurate Voigt

The more_accurate_Voigt keyword can be used to over ride the default Pseudo-Voigt approximation to the Voigt. The more_accurate_Voigt keyword decreases the error (Voigt_approx - Voigt_true) by a factor of around 100. Defining $G$ as the full width at half maximum (FWHM) of a Gaussian and $L$ for the FWHM of a Lorentzian the screen shots below are fits to a range of $G$ convoluted with $L$ (Voigts) with $L$ varying from 0.01 to 0.09 and $G+L=1$.

Fitting to the Voigts using pseudo-Voigts:


Fitting to the Voigts using an accurate calibration:


Note the very small difference-plots for the accurate calibration.
Rescaling the plot vertically to show the whole scan we have:


Note that the difference plot simply appears as a straight line.
The more_accurate_Voigt calibration is accurate and fast. It fits to each true Voigt the following:

```
fit_obj = a1 (2 Sqrt(Ln(2) / Pi) / f1) Exp(-4 Ln(2)(X / f1)^2);
fit_obj = a2 (2 Sqrt(Ln(2) / Pi) / f2) Exp(-4 Ln(2)(X / f2)^2);
fit_obj = a3 (2 / (Pi f3)) / (1 + 4 (X / f3)^2);
fit_obj = a4 (4 / (Pi f4)) / (1 + 4 (X / f4)^2)^2;
```

One thousand sets of $\mathrm{a} 0, \mathrm{a} 1, \mathrm{a} 2, \mathrm{a} 3, \mathrm{f0}, \mathrm{f} 1, \mathrm{f} 2, \mathrm{f} 3$ parameters were determined by fitting to 1000 true Voigts with L varying from 0 to 1 in steps of 0.001 .
The CREATE.INP file in the TEST_EXAMPLESIVOIGT-APPROX $\backslash$ directory creates a true Voigt. It uses the keyword numerical_lor_gauss_conv. The amount of Lorentzian is entered as a number out of a 1000. A number of 500 say would yield a Voigt with a Laurentzian FWHM of 0.5 and a Gaussian FWHM of 0.5 . The generated true Voigt is calculated by numerically convoluting a lor_fwhm with a gauss_fwhm. The generated true Voigt is saved to a file with the name voigtNNNN.xy, where NNNN corresponds to a number between 0 and 1000. The file generated contains 100,000 data points. The step size used in the convolutions is as small as 0.0005 when using a convolution_step of 4 .

TOPAS uses an FFT to actually perform the double summation of the convolution. However, for lor $>500$, the convolution itself comprises an analytical Lorentzian with a Gaussian comprising straight line segments. For lor < 500 then an analytical Gaussian is convoluted with a Lorentzian comprising straight line segments.

- The file FIT-PV.INP fits a pseudo-Voigt to the generated true Voigt.
- The file FIT-MORE.INP fits to the generated true Voigt using the c++ equivalent of fit_obj's.
- The file FIT-OBJ.INP fits fit_obj's to the generated true Voigt.

The difference plot from FIT-PV.INP is in the order of 500 to 1000 times larger than the difference plot from FIT-MORE.INP.

### 2.18User defined rotational matrices

Space group generator - User defined rotational matrices can be added to the file sgrots3.cpp found in the main TA directory.

### 2.19Atomic data files and associated sources

Table 2-1 lists the files read when atomic data is sought. The references refer to the source of the data. In many cases the format of the data file corresponds to the original source format.

Table 2-1 Files and associated sources for atomic data.

| File | Comment |
| :---: | :---: |
| anomdisp.cpp | $f^{\prime}$ and $\mathrm{f}^{\prime \prime}$ for Laboratory X-ray tubes. File is read if there are no associated SSFl**.NFF file or if use_tube_dispersion_coefficients is defined. |
| atmscat.cpp | $\mathrm{f}_{0}$ or Elastic Photon-Atom Scattering, relativistic form factors; data from http://www.esrf.fr/computing/expg/subgroups/theory/DABAX/dabax.html |
| atom_colors.def | Red, Green, Blue (RGB) CPK atom colors from http://www.bio.cmu.edu/Courses/BiochemMols/Periodic/ElemList.htm. Used for assigning colors to atoms when displaying in OpenGL. |
| atom_radius.def | Atomic radii and Covalent radii from http://www.esrf.fr/cgi-bin/periodic. |
| isotopes.txt | Atomic Weights and Isotopic Compositions for All Elements from http://physics.nist.gov/PhysRefData/Compositions/ |
| magdata.dat | Data from GSAS data file via the International tables. Data correction for V entry made by Robert Von Dreele. |
| neutscat.cpp | Neutron scattering lengths from http://www.ccp14.ac.uk/ccp/web-mirrors/neutrons/n-scatter/n-lengths/LIST~1.HTM |
| no_polyhedra.def | Disables drawing of polyhedral for atoms listed. |
| SSF\*.NFF | Anomalous scattering factors $f^{\prime}$ and $f$ " for a range of wavelengths from http://www-cxro.lbl.gov/optical_constants/asf.html <br> The present data is in three columns " $\mathrm{E}(\mathrm{eV}), \mathrm{f} 1, f 2$ " where $\mathrm{f}^{\prime}=\mathrm{f} 1-\mathrm{Z}$ and $\mathrm{f}^{\prime}=\mathrm{f} 2$ and the conversion from wavelength to energy scale is $E(e V)=10^{\wedge} 5 /\left(8.065541^{*}\right.$ Lambda(Ang)). |
| MAClZnn.html | X-Ray Mass Attenuation Coefficients from http://www.nist.gov/pml/data/xraycoef/index.cfm |

### 2.20 Removing Phases during a refinement

The remove_phase keyword (used by the Remove_Phase macro) allows for phase removal during refinement. Typical use is as follows:

```
for strs {
    Remove_Phase(0.3, 1)
}
```

Here a phase is removed if its weight percent is below $0.3 \%$ and if the error in the weight percent is greater than $1 \%$. The phase removal process is executed at the end of a Cycle. Text similar to the following is displayed on removal of a phase:

```
*** Deleting phase: Corundum ***
*** Deleting phase: Zincite ***
... etc...
```

Refinement is terminated when no phase is removed during a Cycle.

### 2.21 Numerical Lorentzian and Gaussian Convolutions

For fundamental and pseudo-Voigt peak types, Lorentzian and Gaussian convolutions are performed analytically during the calculation of the emission profile Voigt. Therefore when lor_fwhm and gauss_fwhm are defined within push_peak and add_pop_1st_2nd_peak keywords they are still calculated at the emission profile level.

## 3 New GUI functionality

### 3.1 Plotting phases above background

By default phases are now plotted on top of back ground where back ground comprises fit_obj's+bkg. The $x d d$ dependent keyword gui_add_bkg and the fit_obj dependent fit_obj_phase can be used to change the defaults, for example,

```
xdd..
    gui_add_bkg !E
    fit_obj-..
        fit_obj_phase !E
```

gui_add_bkg defaults to 1 ; if it's zero then phases are not plotted above back ground.
fit_obj_phase defaults to 1 . If gui_add_bkg =1 then the following is added to phases:
bkg + (and any fit_obj's that has fit_obj_phase =1)
quant\Quant-7.inp shows the use of fit_obj_phase=1 where a fit_obj that is a function of a user_y object, that is supposed to be a phase, is plotted on top of back ground using a dummt_str; the dummy_str checks the status of the fit_obj's fit_obj_phase.

### 3.2 Plotting fit_objs

fit_obj's can be plotted using the following macros:

```
macro Plot_Fit_Obj(p, name)
    {
        dummy_str
            phase name name
            scale= p;
    }
macro Plot_Fit_Obj(name)
    {
        dummy_str
            phase name name
    }
```

See test_exampleslvoigt-approx\fit-obj.inp for example; ie.

```
xdd...
    fit_obj !f1 = ...
    Plot_Fit_Obj(f1, "Fit Obj")
```

Plotting is via a dummy_str and the scale parameter of the dummy_str is set to the name given to the fit_obj, which in this case is $f 1$. At the plotting stage the dummy_str borrows the calculated pattern from the fit_obj.
The scale parameter of the dummy_str has some intelligence built into it such that if scale is not a function of a fit_obj name then it will search the place of the item it is a function of for a calculated pattern. For example, in the following:

```
xdd...
    Plot_Fit_Obj(a, "Fit Obj")
    fit_o\overline{bj = a ...}
        prm a ...
```

the 'a' parameter lives locally to the fit_obj as it is defined after the fit_obj. Defining the scale parameter of the dummy_str in terms of 'a' therefore allows the dummy_str to determine where to find the calculated pattern to display. In this way macros such as the PV macro can be used and plotted without having to define a name for the fit_obj, see test_examples\pvs.inp.

Sometimes the fit_obj has no name and no parameter that belongs to it; instead of naming the fit_obj or rearranging prm definitions the second Plot_Fit_Obj macro can be used:

```
xdd...
    fit obj =
    Plo\overline{t_Fit_Obj("plot previously defined fit_obj")}
```

Here the fit_obj defined prior to Plot_Fit_Obj is plotted.

### 3.3 Display of Normalized SigmaYobs^2

Useful for checking SigmaYobs anomalies from VCT or XYE files; here's an example :


The normalization is as follows:

```
SigmaYobs^2 displayed = SigmaYobs^2 Sum[ Yobs ] / Sum[ SigmaYobs^2]
```

This puts the display of SigmaYobs^2 on a similar scale to Yobs. For normal x-ray data
SigmaYobs=Sqrt(Yobs) and hence nothing is done as the displayed plot would simply be equal to Yobs. On some data sets, TOF for example, the magnitude of SigmaYobs can be small; thus when refining on multiple data sets from different sources the weighting schemes may need to be modified in order to give the desired weight to the data sets. The option for display is as follows::


### 3.4 Cumulative Chi2

A kernel operation that results in the following graphical display:


- Uses the weighting from the kernel which can be user defined or otherwise.
- SigmaYobs is used in the weighting if it exists.
- Prior to graphical display it is scaled to have the same maximum intensity as the maximum of Yobs.
- Data is obtained from the kernel and thus excluded regions are ignored as shown in the plot above.
- Tabs for Cumulative Chi2 has been included in the appropriate GUI tabs as seen in the following:



### 3.5 Correlation Matrix display

A Correlation matrix window activated from the Fit Dialog; it operates in Launch or GUI modes. Example output is as follows:


Both the A-matrix and the correlation matrix include penalties/restraints depending on whether do_errors_include_penalties and/or do_errors_include_restraints are defined. The display of the matrix can be zoomed using Ctrl-MouseWheel, for example:


MouseMove over the correlation matrix displays a Hint comprising the corresponding parameter names, values and errors. Left Mouse button down and dragging translates the matrix.

### 3.6 Fading a structure

The intensity of atom colours displayed in OpenGL can be adjusted using the Fade spin button of the OpenGL options grid; for example:


### 3.7 Normals Plot

An OpenGL plot of lattice plane Normals with the lengths of the Normals defined by the keyword normals_plot. For example:

```
normals plot = Abs(H * K + L^2) + 1;
    normals_plot_min_d . 3
```

normals_plot_min_d is optional; small values (ie. 0.1) could lead to millions of points and users could blow up their computers. Here's some output from the test example CLAY.INP:


## 4 tc-inps.bat

The batch file TC-INPS.BAT found in the main TOPAS directory runs TC.EXE through around 80 test examples. Here's a snippet from TC-INPS.BAT

```
tc test_examples\mag\mag "#define CREATE_"
tc test_examples\mag\mag
tc test_examples\occ-merge "macro aac$ { iters 3000 }"
```

The macro called "aac\$" instructs the program to place what's in aac\$ at the bottom of the INP file. TCINPS.BAT takes 3 to 5 minutes to run. Output can be placed into a file as follows:

```
tc-inps.bat > some_file
```

some_file will contain around 4000 lines of output.

## 5 Interface Specific

### 5.1 Improvements to the Grid

- Data can now be sorted by Double Clicking on Column Headings. Sorting alternates between ascending and descending order. On leaving a particular grid the column most recently sorted is remembered. On re-entry of that particular grid the data is again sorted according to the saved state. A small < or > sign is displayed to the left of the Column heading name. Sorting works for all grids that display data with rows that are similar in Type; ie. Peak data, sites etc.... Val and Error columns are sorted numerically. Hkls, $\mathrm{F}^{\wedge} 2$ and other obvious numeric columns are also sorted numerically. However, Min and Max are sorted using strings as they can be equations and hence their fields are strings.
- CTRL-MouseWheel zooms/un-zooms the text of a grid
- MouseDownMouseMove for Panning.


### 5.2 Mouse operation in OpenGL Graphics

First some definitions

- LMB = Left Mouse Button
- $\quad$ RMB $=$ Right Mouse Button
- MID = Mouse Wheel or Middle button on Laptops
- $\mathrm{MM}=$ Mouse Moving
- $\mathrm{WM}=$ Wheel moving
- LMB-D = Left Mouse Button Down
- RMB-D = Right Mouse Button Down
- MW-D = Mouse Wheel Down
- For example, LMB-D- MM is simply dragging with the LMD Image rotation/translation operations are:
- LMB-D- MM rotates the image.
- LMB-D- MM and quick release initiates continuous rotation.
- LMD-D-MM on the first $10 \%$ of the viewport from the left or the last $10 \%$ from the right rotates around an axis perpendicular to the screen. This is another way of doing what Shift-LMB-D-MM does but without the need for keyboard input. $10 \%$ seems a good amount as it does not seem to interfere with normal rotation.
- MW zooms in addition to the usual RMB-D-MM.
- MID-D-MM translates the image in the plane of the screen.

Images are rotated around the centre of gravity (or centre of unit cell) unless there's a change using the RMB-D options.

## 6 Kernel Specific

### 6.1 New keywords

chi2
chk_for_best
current_peak_max_x
current_peak_min_x
def
mag_space_group
mg
mlx
mly
mlz

```
del_approx
do_errors_include_restraints
do errors include penalties
element_weight_percent
element_weight_percent_known
elemental_composition
existing_prm
f0
f0_f1_f11_atom
f1
f11
ft_conv
ft_x_axis_range
ft_min
fit_obj_phase
fo_transform_X
fn
generate_these
generate_name_append
gui_add_bkg
layer
lpsd_beam_spill_correct_intensity
mag_atom_out
mag_only
mag_only_for_mag_sites
tangent_tiny
xdd_sum
```


### 6.2 New Test Examples

| test_exampleslwppm |  |
| :--- | ---: |
| compare-1.inp |  |
| cube-In-normal-1.inp |  |
| gamma-fit-obj.inp |  |
| gamma.inp |  |
| In-normal-1.inp |  |
| sphere-fit-obj.inp |  |
| sphere-gamma-compare-1.inp |  |
| sphere-gamma-compare-2.inp |  |
| spere-gamma-compare-3.inp |  |
| super-lorentzian.inp |  |
| s-sphere-1.inp |  |
| test_exampleslftl |  |
| create-voigt.inp | Laun |
| alvo4a.inp | Laun |
| gaussian.inp | Laun |
| lorentzian.inp | Laun |
| voigt.inp | Laun |
| test_exampleslsingle-crystall |  |
| ylidma.inp | Launch |
| gebaa.inp | Launch |
| ae1-adps.inp | Launch |
| ae1-auto.inp | Launch |
| ae1-manual.inp | Launch |
| ae1-approx-a.inp | Launch |
| ae5-auto.inp | Launch |
| ae14-approx-a.inp | Launch |
| pn_02_2.inp | Launch |

compare-1.inp
cube-In-normal-1.inp
gamma-fit-obj.inp
gamma.inp
sphere-fit-obj.inp
sphere-gamma-compare-1.inp
sphere-gamma-compare-2.inp
sphere-gamma-compare-3.inp
super-lorentzian.inp
s -sphere-1.inp
create-voigt
alvo4a.inp
gaussian.inp
Iorentzian.inp voigt.inp

Launch
Launch
Launch
Launch
Launch
unch test_examples\single-crystal\

Launch
Launch
Launch Launch Launch Launch Launch Launch Launch Launch Launch

```
modify_peak
```

modify_peak
modify_peak_apply_before_convolutions
modify_peak_apply_before_convolutions
modify_peak_eqn
modify_peak_eqn
more_accurate_Voigt
more_accurate_Voigt
no_f11
no_f11
no_inline
no_inline
normals_plot
normals_plot
normals_plot_min_d
normals_plot_min_d
numerical_lor_gauss_conv
numerical_lor_gauss_conv
numerical_lor_ymin_on_ymax
numerical_lor_ymin_on_ymax
out_prm_vals_dependents_filter
out_prm_vals_dependents_filter
out_refinement_stats
out_refinement_stats
remove_phase
remove_phase
report_on_str
report_on_str
return
return
save_best_chi2
save_best_chi2
scale_phase_X
scale_phase_X
stack
stack
sx
sx
sy
sy
sz
sz
user_y
user_y
WPPM_ft_conv
WPPM_ft_conv
WPPM_L_max
WPPM_L_max
WPPM_th2_range
WPPM_th2_range
WPPM_correct_Is

```
WPPM_correct_Is
```

```
test_examples\tof
    tof_balzar_br1.inp Launch
    tof balzar sh1.inp Launch
    tof_bank2_1.inp Launch
    tof_bank2_2.inp Launch
test_examples\user_y\
    cpd1e.inp
    Launch
    user_y.inp Launch
test_examples\\p-search\
    lp-search-cimetidine.pro
        GUI
    lp-search-pbso4.pro GUI
    lp-search-pbso4.inp
        GUI-Launch
test_examples\absorption-edge\
```

Ni-LaB6.inp
Al2O3-pam.inp
Spinel-pam.inp
Rutile-Anatase.inp
Rutile-Anatase-Ni.inp
est_examples\voigt-approx\}
create.inp Launch
fit-more.inp Launch
fit-pv.inp Launch
fit-obj.inp Launch
test_examples\f0-f1-f11\}
xray-powder.inp Launch
tof.inp Launch
test_examples\penalties-restraints\}
rosenbrock-10.inp Launch
rosenbrock-10-restraint.inp Launch
hock.inp Launch
rastrigin.inp Launch
test_examples\mag\}
mag.inp Launch
mag-2.inp Launch
occ-merge.inp Launch
mag-only.inp Launch
maglamno3_magnetic.inp Launch
test_examples\quant $\backslash$
quant-1.inp Launch
quant-2.inp Launch
quant-3.inp Launch
quant-4.inp Launch
quant-5.inp Launch
quant-6.inp Launch
quant-7.inp Launch
quant-7-create.inp Launch
quant-8.inp
zro2-restraint-wt.inp
zro2-restraint-xrf-zr.inp
test_examples\rigid\}
rigidb.inp Launch
rigida-1.inp Launch
rigida-2.inp Launch
test_examples\rigid-errors\}
aniline_I_100K_x.inp Launch
Aniline_I_8kbar_n.inp Launch
test_examples\stacking-faults\}
kaolinite-layer.inp Launch

| debye-old.inp <br> debye-new.inp <br> test_exampleslfunctions <br> alvo4-fn.inp <br> alvo4-normal.inp <br> fn-test.inp | Launch <br> Launch |
| :--- | :--- |
| test_examples\laue\} $\begin{array} { l } { \text { laue.inp } } \end{array}$ $&{$ Launch  <br>  test_examples\k-factor\  <br>  k-factor.inp $} \\ {$ k-factor.pro $} &{\text { Launch }}$ |  |

### 6.3 New Equation Functions

```
Cosh
Erf Approx
Erfc_Approx
Error
Gamma_Ln_Approx
Gamma Approx
Get_Element_Weight
Ln_Normal_x_at_CD
Ob\overline{j There}
Prm_There
Round
Sinh
Tanh
```

Example output from Round:

```
prm = Round(.1); : 0.00000
prm = Round(.5); : 0.00000
prm = Round(1.6); : 2.00000
prm = Round(-.1); : 0.00000
prm = Round(-.5); : 0.00000
prm = Round(-1.6); : -2.00000
```


## 7 Pre-Processor

### 7.1 New Macros

```
Bkg_Straight_Line
Cu6_Ni_Edge
EP_Absorption_Edge_Correction
EP_Absorption_Edge_Correction_Eqn
LP_Factor_X
MM_Cartesian_Display
MM_Cartesian_Refine
Out_CIF_Bonds_Angles
Remove_Phase
Robust_Refinement
```


### 7.2Defining unique parameters within macros

\#m_unique \$string assigns a unique parameter name to \$string within a macro. This allows new unique parameters to be defined within macros without the worry of name clashes. In the example:

```
macro Some_macro(b) { prm #m_unique a = Cos(Th); }
```

' $a$ ' is assigned a unique parameter name and it has the scope of the macro body text. The Robust_Refinement and TCHZ_Peak_Type macros are good examples of its use, where for example, the former is defined as:

```
macro Robust_Refinement
{
    ' Rescale peaks according to robust refinement algorithm
    prm #m_unique test = Get(r_exp);
    prm #m_unique N = 1 / test^2;
    prm #m_unique p0 = 0.40007404;
    prm #m_unique p1 = -2.5949286;
    prm #m_unique p2 = 4.3513542;
    prm #m_unique p3 = -1.7400101;
    prm #m_unique p4 = 3.6140845e-1;
    prm #m_unique p5 = -4.45247609e-2;
    prm #m_unique p6 = 3.5986364e-3;
    prm #m_unique p7 = -1.8328008e-4;
    prm #m_unique p8 = 5.7937184e-6;
    prm #m_unique p9 = -1.035303e-7;
    prm #m_unique p10 = 7.9903166e-10;
    prm #m_unique t = (Yobs - Ycalc) / SigmaYobs;
    weightīng = If( t < 0.8,
        N / Max(SigmaYobs^2, 1),
        If( t < 21, N ((()(()(((p10 t + p9) t + p8) t + p7)
        t + p6) t + p5) t + p4) t + p3)
        t + p2) t + p1) t + p0) / (Yobs - Ycalc)^2,
        N (2.0131 Ln(t) + 3.9183) / (Yobs - Ycalc)^2));
    recal_weighting_on_iter
}
```


### 7.3 Superfluous parentheses and the ' $\&$ ' Type for macros and its arguments

The pre-processor is an un-typed language meaning that it knows nothing about the type of text passed to macro arguments. This has great flexibility but there can be drawbacks; for example, the following:

```
macro divide(a, b) { a / b }
prm e = divide(a+b, c-d);
```

expands to the unintended result of:

```
prm e = a + b / c - d;
```

The writer of the macro could solve this problem by rewriting the macro with parentheses:

```
macro divide(a, b) { (a) /( b) }
```

Alternatively the \& Type can be used for macros that expect equation type arguments. Defining the macro with ' $\&$ ' before the arguments as in:

```
macro divide(& a, & b) { a / b }
prm e = divide(a+b, c-d);
```

instructs the pre-processor that the argument is of an equation Type and a check is made to determine whether the argument needs parentheses. This results in the correct expansion of:

```
prm e = (a+b) / (c-d);
```

Even with \& types used for arguments, the following:

```
macro divide(& a, & b) { a / b }
prm e = divide(a+b, c-d)^2;
```

expands to the unintended:

```
prm e = (a + b) / (c - d)^2;
```

The writer of the macro could again rewrite the macro to include more parentheses:

```
macro divide(a, b) { ((a) / (b)) }
```

Or, define the expansion of the macro itself to have an \& Type by placing the \& character before the macro name itself as follows:

```
macro & divide(& a, & b) { a / b }
```

Expansion of prm $e=\operatorname{divide}(a+b, c-d)^{\wedge} 2$ now becomes the intended:

```
prm e = ((a + b) / (c - d) )^2;
```

With the use of the \& Type, macros such as Ramp defined in Version 4 as:

```
macro Ramp(x1, x2, n)
{
    ((x1) + ((x2)-(x1)) Mod(Cycle_Iter, (n)) / ((n)-1))
}
```

can now be written with less parentheses as follows:

```
macro & Ramp(& x1, & x2, & n)
{
    x1 + (x2-x1) Mod(Cycle_Iter, n) / (n-1)
}
```


### 7.4 Pre-processor equations and \#prm, \#if, \#elseif, \#out, \#m_if, \#m_elseif, \#m_out

Pre-processor parameters, called hash parameters, can be defined by placing a \# before the text prm. \#prm's can be a function of other \#prm's and they can be used in \#if, \#elseif, \#m_if and \#m_elseif preprocessor statements. \#prm's are only evaluated at the pre-processor stage of loading INP files (see test_examples\hash_prm.inp); they are therefore unknown to the kernel and are totally separate to parameters defined using prm. Pre-processed output can be found in the TOPAS.LOG file when running TA.EXE or TC.LOG when running TC.EXE.

The \#out and \#m_out allows pre-processor \#prm's values, which can be strings or numbers, to be placed into the pre-processed text. For example:

```
#prm a = Constant(Rand(0,1));
#out a
```

will output a random number between 0 and 1 into the pre-processed file at the position of \#out. INP files can therefore be manipulated with \#prm's and \#if statements with a means of identifying the manipulation carried out.

The following:

```
macro Ex1(a)
{
    #m_if a == "b";
        Yes b
    #m_elseif a == "c";
        Yes c
    #m_endif
}
Ex1("b")
```

expands to:
Yes b
In the following:

```
#prm ran = Constant(Rand(0,1));
#if ran < 0.5;
    view_structure
#endif
#if ran < 0.5;
    view_structure
#endif
#if ran < 0.5;
    view_structure
#endif
```

each call to 'ran' in the \#if statements would return the same value because of the use of Constant.
More complicated INP file manipulation is shown in the following:

```
#prm space_group_number = 4;
#if And(space_group_number >= 75, space_group_number <= 142);
#elseif And(space_group_number >= 16, space_group_number <= 74);
#endif
```


## 8 Keywords removed

```
swap_sites
try_site_patterns
break if been there
hkl_Is_frrom_h\overline{kl4}
do_processes
```


## 9 References

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